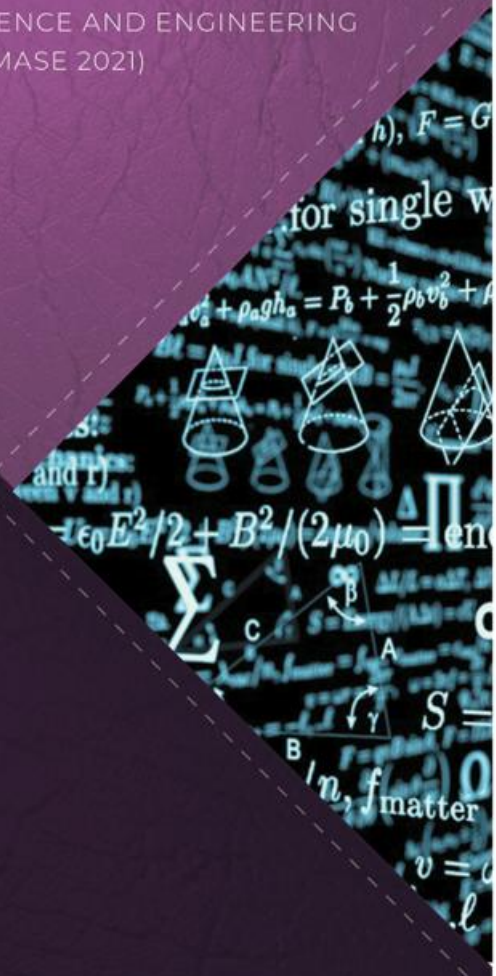
ICMASE 2021

II. INTERNATIONAL CONFERENCE ON MATHEMATICS

AND ITS APPLICATIONS IN SCIENCE AND ENGINEERING (ICMASE 2021)



ABSTRACT BOOK

VNiVERSiDAD D SALAMANCA
CAMPUS OF INTERNATIONAL EXCELLENCE

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Preface

This abstract booklet includes the abstracts of the papers that have been presented at II. International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2021) which is held in University of Salamanca, Spain between 1-2 July, 2021, via online because of COVID-19 pandemic. The aim of this conference is to exchange ideas, discuss developments in mathematics, develop collaborations and interact with professionals and researchers from all over the world about some of the following interesting topics: Functional Analysis, Approximation Theory, Real Analysis, Complex Analysis, Harmonic and non-Harmonic Analysis, Applied Analysis, Numerical Analysis, Geometry, Topology and Algebra, Modern Methods in Summability and Approximation, Operator Theory, Fixed Point Theory and Applications, Sequence Spaces and Matrix Transformation, Modern Methods in Summability and Approximation, Spectral Theory and Diferantial Operators, Boundary Value Problems, Ordinary and Partial Differential Equations, Discontinuous Differential Equations, Convex Analysis and its Applications, Optimization and its Application, Mathematics Education, Applications on Variable Exponent Lebesgue Spaces, Applications on Differential Equations and Partial Differential Equations, Fourier Analysis, Wavelet and Harmonic Analysis Methods in Function Spaces, Applications on Computer Engineering, Flow Dynamics. However, the talks are not restricted to these subjects.

Thanks to all committee members.

We wish everyone a fruitful conference and pleasant memories from ICMASE 2021.

Prof. Dr. Araceli QUEIRUGA-DIOS,

Assoc. Prof. Dr. Fatih YILMAZ

Chairs, ICMASE 2021

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ON A FRACTIONAL LAZER–MCKENNA CONJECTURE WITH SUBLINEAR NONLINEARITY

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ABSTRACT

Let $(-\Delta)^s$ be the classical fractional Laplacian operator of order $2s$ defined for any function
$u : \mathbb{R}^N \longrightarrow \mathbb{R}$ belonging to the Schwartz space $\mathcal{S}(\mathbb{R}^N)$ of rapidly decaying C^∞ function in $\mathbb{R}^N$ by:

$$(-\Delta)^s u(x) := a_{N,s} \text{ P.V. } \int_{\mathbb{R}^N} \frac{u(x)-u(y)}{|x-y|^{N+2s}}\, dy,\ s \in (0,1), \tag{1}$$

where

$$a_{N,s} := 2^{2s-1}\pi^{-\frac{N}{2}} \frac{\Gamma(\frac{N+2s}{2})}{|\Gamma(-s)|},$$

is the normalization constant to have the identity

$$(-\Delta)^s u = \mathcal{F}^{-1}(|\xi|^{2s}\mathcal{F}u),\ \xi \in \mathbb{R}^N, s \in (0,1),$$

with $\mathcal{F}$ being the Fourier transform.

Fractional operators have gained increasing popularity in recent years. This is due to both the natural mathematical interest of such subject and to the various applications that they allow. In fact, fractional calculus can be of good use if one wants to develop more sophisticated mathematical models that can accurately describe complex anomalous systems. Therefore, nonlocal pseudodifferential operators such as $(-\Delta)^s$ are naturally involved in population dynamics, continuum mechanics, game theory and other phenomena, as the infinitesimal generators of Lévy-type stochastical processes.
We consider the following nonlinear fractional Dirichlet problem :

$$\begin{cases} (-\Delta)^s u &= f(u) + t\phi & \text{in } \Omega, \\ u &= 0 & \text{in } \mathbb{R}^N \setminus \Omega \end{cases} \tag{2}$$

where $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary, $N > 2s$, $s \in (0,1)$, $t \in \mathbb{R}$, $\phi \in L^2(\Omega)$ and $f \in C(\mathbb{R}, \mathbb{R})$ satisfies the condition below, besides other conditions that will be timely intoduced when we proceed:

$$\lim_{\xi \to \pm\infty} \frac{f(\xi)}{\xi} = f_\pm \in \mathbb{R} \quad \text{and} \quad f_- < f_+.$$

The natural funtional setting to study Dirichlet-type boundary value problems such as problem (2) is provided by the fractional Sobolev spaces $W_0^{s,p}(\Omega)$ defined, for $1 \leq p < \infty$ and $s \in (0,1)$ by

$$W_0^{s,p}(\Omega) := \left\{ u \in W^{s,p}(\mathbb{R}^N) : u = 0 \text{ in } \mathbb{R}^N \setminus \Omega \right\}.$$

where

$$W^{s,p}(\mathbb{R}^N) := \left\{ u \in L^2(\mathbb{R}^N) : \int_{\mathbb{R}^N \times \mathbb{R}^N} \frac{|u(x) - u(y)|^p}{|x-y|^{N+ps}} dxdy < \infty \right\}.$$

We denote by $(\lambda_k)_{k \in \mathbb{N}^\star}$ the sequence of eigenvalues of $(-\Delta)^s$ on $H_0^s(\Omega) := W_0^{s,2}(\Omega)$. We also denote by ϕ_k the eigenfunction corresponding to the k-th eigenvalue λ_k and which we suppose, is normalized in $L^2(\mathbb{R}^N)$, that is $\|\phi_k\|_{L^2(\Omega)} = 1$. It is well known that the first eigenvalue λ_1 is simple and that the first eigenfuntion ϕ_1 is stricly positive in Ω.
We are interested in the case where $\phi = \phi_1$, f belongs to $C^1(\mathbb{R}, \mathbb{R})$ and satifies the following Ambrosetti Prodi type condition :

$$f_- < \lambda_1 < \lambda_2 < f_+.$$

We say that the nonlinearity f crosses the first two eigenvalues.
In this case, we aim to prove, the existence of a real constant $t_0 \in \mathbb{R}$ such that if $t < t_0$, problem (2) has at least four different weak solutions, and that would be a generalization of the multiplicity results proved in the old litterature By S. Solimini in [5] and by H. Hofer in [2], for the local version of problem (2). This could, also, be a good start to answer some of the questions given by Lazer, Mckenna and Pellico in [3]

***K*eywords** Fractional Laplacian · Lazer–McKenna conjecture · Multiplicity of solutions · Topological degree theory

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LİFT CURVES AND RULED SURFACES

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ABSTRACT

The concepts of ruled surface are, firstly, studied by G.Monge. In literature, any ruled surface occurs as a result of the continuously movement of a straight line along any curve, which are called the director and the base curve of the surface, respectively. Moreover, ruled surfaces have significiant application areas such as CAD, kinematics , architecture, etc. The definition of natural lift curve is,firstly, given in J.A Thorpe's book. According to definition, the natural lift curve is obtained as a curve generated by combining the end points of the the unit tanget vectors at each point of a curve.In this study,firstly, the Darboux vectors $\overline{W}$ of the natural lift curve $\overline{\alpha}$ for the given curve α are calculated in terms of the types of curves in Minkowski space. Secondly, striction lines and distribution parameters of ruled surface pairs generated by the Darboux vectors have been computed. Finally, the shape operator, Gaussian curvature and the first and second fundamental forms have been denoted.

***K*eywords** Lift Curves · Ruled Surface · Darboux Vector

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THE HIDDEN ROLE OF THE PRE-SYMPTOMATIC INDIVIDUALS IN THE TRANSMISSION DYNAMICS OF COVID-19

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ABSTRACT

A mathematical model with three different routes of transmission, namely, asymptomatic, pre-symptomatic and symptomatic transmissions, has been proposed and analyzed to investigate the role of pre-symptomatic individuals in the transmission dynamics of COVID-19 outbreak. Using the next generation matrix method, the basic reproduction number $R0$ has been derived and then sensitivity analysis of the proposed model is presented. Existence and stability analysis of disease free and endemic equilibrium points have been discussed. Numerical simulations to demonstrate the effect of some model parameters related to pre-symptomatic transmission on the disease transmission dynamics have been carried out.

***K*eywords** COVID-19 · pre-symptomatic individuals · basic reproduction number · stability analysis · sensitivity analysis

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ON TRIANGULAR MATROIDS INDUCED BY n_3-CONFIGURATIONS

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ABSTRACT

A triangular matroid is a rank-3 matroid whose ground set consists of the points of an n_3-configuration and whose bases are the point triples corresponding to non-triangles within the configuration. Raney previously enumerated the n_3-configurations which induce triangular matroids for $7 \leq n \leq 15$. In this work, the enumeration is extended to configurations having up to 18 points. Several examples of such configurations and their symmetry groups are presented, as well as geometric representations of the triangular matroids induced by these configurations.

***K*eywords** classification · configuration · matroid · triangle

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APOLLONIUS PROBLEM ON THE NUMBER OF NORMALS PASSING THROUGH A POINT OF A CONIC

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ABSTRACT

We study a variant of Apollonius problem on determining the number of normals to an ellipse passing through a given point. The origin of the problem is Book V of Apollonius' Conics. We mention also the case of hyperbolas and parabolas. We study the following problem: Let A be a general point on the plane of the ellipse with equation $\frac{x^2}{a^2}+\frac{a^2}{b^2}=1$, where $a>b>0$ How many points B, different from A, do exist on this ellipse, such that the line AB is perpendicular to the tangent line of this ellipse at the point B? We denote this number by $n(A)$. We do not exclude the case where the point A is on the given ellipse. In fact the main aim of the present paper is to study this case in more detail.

Firstly, we suppose that the point A is not on the ellipse. In this variant the problem (Apollonius Problem) was considered in [4, §13]; [3] p. 71 (problem 13), p. 257-258 (solution); [1, Section 17.7.4]. For historical background of this problem see [5, Ch. VII] and [2, Ch. 12]. We will present here a new solution using elementary calculus. After this we find intersection points of the given ellipse and its astroida. The obtained points surprisingly have elegant coordinates. These points completely solve the problem for the points of the given ellipse.

***K*eywords** Apollonius Problem, Conics, Normals

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MATHEMATICAL ANALYSIS OF A ZIKA MODEL WITH RESERVOIRS AND HUMAN MOVEMENT.

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ABSTRACT

Recently, Zika virus infection has become a perilous danger for the human society. Zika is a flavivirus transmitted to humans through either the bites of infected mosquitoes or sexual transmission. A mathematical model for Zika virus is proposed describing the spread of the disease in three interacting populations, namely, human, vector (mosquitoes) and non-human primate (monkeys) inhabiting forests area. It is assumed that Zika virus spreads within non-human primate population, which in turn acts as a reservoir of infection, and then transmitted to the human population through infected mosquitoes. The proposed model incorporates vertical transmission and direct transmission in all populations. Human movement between rural and forest areas has been also considered. The proposed model has been first normalized. The normalized model has been then fully analyzed both qualitatively and quantitatively. The mathematical analysis includes positivity and boundness of solutions, derivation of the basic reproduction number R_0 using the next generation matrix method, sensitivity analysis, existence and stability analysis of all equilibria and bifurcation analysis. Finally, numerical simulations have been carried out to illustrate the obtained theortical results and to demonstrate the effect of some model parameters related to the different routes of disease transmission.

***K*eywords** ZIKV · Basic Reproduction Number · Stability Analysis · Sensitivity Analysis

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ON THIRD ORDER BRONZE FIBONACCI NUMBERS

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ABSTRACT

In this study we investigate the generalized third order Bronze Fibonacci sequence and three specific sequences which are derived from its De-Moivre Type Identities. We give the generating functions, Binet's Formulas, Cassini's Identity and matrix representation of these sequences. Moreover, we obtain some interesting identities related with third order Bronze Fibonacci sequences. Finally, we give an application of our results to encryption theory, a third order Bronze Fibonacci encryption and decryption algorithm.

***K*eywords** Bronze Fibonacci sequence · De-Moivre Type Identities · Binet's Formulas

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HYPERSURFACE FAMILIES CONSTRUCTED BY ISOPARAMETRIC SMARANDACHE CURVES OF AN ASYMPTOTIC CURVE IN G_4

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ABSTRACT

A curve $\delta(\upsilon)$ on a hypersurface $\Omega(\upsilon,\vartheta,\zeta)$ is geodesic if and only if normal $N(\upsilon)$ of the curve $\delta(\upsilon)$ and normal $\eta(\upsilon,\vartheta_0,\zeta_0)$ of the hypersurface $\Omega(\upsilon,\vartheta,\zeta)$ at any point on the curve $\delta(\upsilon)$ are parallel to each other. If the curve is both geodesic and parameter (isoparametric) curve on the hypersurface Ω, then it is called isogeodesic on the hypersurface Ω. In [6], by utilizing the Frenet trihedron frame along the given geodesic, the authors have expressed the surface pencil as a linear combination of the components of this local coordinate frame and derived the necessary and sufficient conditions for the coefficients to satisfy both the geodesic and the isoparametric requirements. In 2008, the generalization of the Wangs' assumption to more general marching-scale functions has been given in [3].

Furthermore, an asymptotic curve, which is an important topic for differential geometers and used in astronomy, astrophysics and CAD in architecture, is a curve α in a regular surface $M \subset R^3$ for which the normal curvature vanishes in the direction α'. And a curve $\delta(\upsilon)$ on the hypersurface $\Omega(\upsilon,\vartheta,\zeta)$ is asymptotic if and only if normal $N(\upsilon)$ of the curve $\delta(\upsilon)$ and normal $\eta(\upsilon,\vartheta_0,\zeta_0)$ of the hypersurface $\Omega(\upsilon,\vartheta,\zeta)$ at any point on the curve $\delta(\upsilon)$ are perpendicular. If the curve is both asymptotic and parameter (isoparametric) curve on Ω, then it is called isoasymptotic on the hypersurface Ω.

In the present study, we deal with the hypersurface families with Smarandache curves in 4-dimensional Galilean space G_4. In this context, we construct eleven types of hypersurface families with different Smarandache curves and we state a main theorem with the aid of a table which contains the conditions for the curve to be asymptotic where Smarandache curves of the curve are isoparametric on hypersurfaces. Also, we give an example for these hypersurface families.

***K*eywords** Asymptotic curve, Smarandache curve and Hypersurface family.

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ON INVOLUTE CURVES AND B-LIFT CURVES IN MINKOWSKI 3-SPACE

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ABSTRACT

In this study, we introduce a new curve in Minkowski 3-space which called B-lift curve and we obtain the Frenet operators of the B-lift curve. Moreover, we examine the correpondence of Frenet vectors between the involute and the B-Lift curve. Finally, we give some examples on these results.

***K*eywords** B-Lift curve · Involute Curve · Frenet Vectors

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A NEW APPROACH FOR SOLVING FRACTIONAL DELAY DIFFERENTIAL EQUATIONS

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ABSTRACT

In this study, we give a numerical aprroach for solving the fractional delay differential equation

$$\sum_{k=1}^{m} P_k(x) D_*^{k\alpha} y(x) + \sum_{r=0}^{m} y(\lambda_r x + \beta_r) = f(x) \qquad (1)$$

under the condition

$$D_*^i y(c) = \mu_i, \quad i = 0, 1, \cdots, n-1, \quad a \leq c \leq b \qquad (2)$$

where λ_r, μ_i and β_r are appropriate constants.
The delay differential equation plays an important role in pure and applied mathematics such as finance an bioenergy [1,2]. We investigate the approximate solution of Eq(1) under the conditions Eq(2) with the fractional Taylor series [3] by using Taylor collocation method[4].

***K*eywords** Fractional delay differential equation · Taylor series · Collocation method

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UPPER BOUNDS ON THE ENERGY OF GRAPHS IN TERMS OF MATCHING NUMBER

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ABSTRACT

The energy of a graph G, $\mathcal{E}(G)$, is the sum of absolute values of the eigenvalues of its adjacency matrix. The matching number $\mu(G)$ is the number of edges in a maximum matching. In this paper, for a connected graph G of order n with largest vertex degree $\Delta \geq 6$ we present two new upper bounds for the energy of a graph: $\mathcal{E}(G) \leq (n-1)\sqrt{\Delta}$ and $\mathcal{E}(G) \leq 2\mu(G)\sqrt{\Delta}$. The latter one improves recently obtained bound

$$\mathcal{E}(G) \leq \begin{cases} 2\mu(G)\sqrt{2\Delta_e+1}, & \text{if } \Delta_e \text{ is even;} \\ \mu(G)(\sqrt{a+2\sqrt{a}}+\sqrt{a-2\sqrt{a}}), & \text{otherwise,} \end{cases}$$

where Δ_e stands for the largest edge degree and $a = 2(\Delta_e+1)$. We also present a short proof of this result and several open problems.

This is a joint work with Abdullah Alazemi (Kuwait University, Kuwait) and Saieed Akbari (Sharif University, Tehran, Iran).

***K*eywords** Graph Energy, Matching Number

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DE MOIVRE-TYPE IDENTITIES FOR THE JACOBSTHAL NUMBERS

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ABSTRACT

The main aim of this study is to obtain De Moivre-type identities for Jacobsthal numbers. Also, this paper presents a method for constructing the second order Jacobsthal and Jacobsthal-Lucas numbers and the third order Jacobsthal and Jacobsthal-Lucas numbers. Moreover, we give some interesting identities, such as Binet's formulas for some specific third order Jacobsthal numbers that we derive from De Moivre-type identities.

***K*eywords** De Moivre-type identity · Jacobsthal numbers · Generating functions · Binet's formula

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DE-MOIVRE'S AND EULER FORMULAS FOR MATRICES OF HYBRID NUMBERS

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ABSTRACT

It is known that the hybrid numbers are generalizations of complex, hyperbolic and dual numbers. Recently, they have attracted the attention of many scientists. At this paper, we provide the Euler's and De Moivre's formulas for the 4×4 matrices associated with hybrid numbers by using trigonometric identities.

***K*eywords** Hybrid Numbers · Matrix Representation · Euler's and De-Moivre's formulas

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SOME FIXED POINT RESULTS FOR ALMOST (α, q) −CONVEX CONTRACTIVE MAPPINGS VIA SIMULATION FUNCTION

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ABSTRACT

In this study we introduce almost $(\alpha, q)-$convex contraction via simulation function. We give an example and prove some fixed point results for such mappings in metric spaces. As a conclusion, our results generalizes various contractions on metric space.

***K*eywords** Fixed Point · convex contraction · simulation function

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APPROXIMATION BY KANTOROVICH VARIANT OF MAX-MIN OPERATORS WITH APPLICATIONS TO DIGITAL IMAGE PROCESSING

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ABSTRACT

Nonlinear approximating operators have always been attractive for the mathematicians especially who are concerned with approximation theory, since they may achieve better approximations results when compared to their linear counterparts [3, 2, 1, 2, 4]. For this purpose, Bede et al. construct pseudo-linear operators by changing the algebra of summation and multiplication in the discrete operators with maximum and minimum operations respectively [2]. For further results about the pseudo linear operators, we refer [5, 9]. Then, Coroianu and Gal examined the Kantorovich type max-product operators in [5], while on the other hand, Duman and Gökçer in [10, 11] obtained a general approximation results for pseudo linear operators of max-min kind. We should remind that these operators are quiet effective in fuzzy logic [7]. Although there are a lot of researches about max-product type operators, in the literature, there are only a few papers for max-min kind ones. Initiating from the above studies, in this work, we construct a general form for Kantorovich-type max-min operators in both univariate and multivariate settings. Note that, by the help of the Kantorovich variant of these operators, it is possible to have more information about the given function since we replace the sampled values of the function f with an average value of it in a neighborhood of this sampled point. Then we also study the rate of approximation by using Hölder continuous functions and modulus of continuity. Later, as a special case we prove that our operator includes Bernstein, Shepard and many other different operators. Furthermore, shape preserving properties of kantorovich type max-min Bernstein operators will be investigated, which are important in fuzzy logic. Then we use them in approximating to fuzzy continuous functions which are quasi-concave. Moreover, we illustrate our approximations by plotting them in figures. Finally, as a real life problem, we

consider the digital image processing and apply our approach to obtain increased resolution images.

Acknowledgement. This study is supported by TÜBİTAK 3501 Career Development Program, Project ID: 119F262.

***K*eywords** Max-Min Operators · Kantorovich Operators · Digital Image Processing

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APPROXIMATION BY MATRIX TRANSFORMS IN MORREY-SMIRNOV CLASS

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ABSTRACT

In this work, we investigate the error of approximation of trigonometric polynomials using matrix transform based on Nörlund Method and prove some trigonometric approximation theorems with degree of $(n^{-\gamma})$ in the subclasses of Morrey spaces. Also, we study approximation properties of matrix transforms constructed via the Faber series in the Morrey-Smirnov classes of analytic functions. Our main results are the following.

Theorem 1 *Let* $f \in Lip_{\alpha,p}(\mathbb{T},\gamma)$, $0 < \alpha \leq 2$, $1 < p < \infty$, $0 < \gamma < 1$ *and* $A = (a_{n,k})$ *be a lower triangular matrix with* $\left|S_n^{(A)} - 1\right| = O\left(n^{-\gamma}\right)$. *If one of the following conditions*

$$\begin{aligned} (i)\ \ (a_{n,k}) &\in AMDUMS \\ (ii)\ \ (a_{n,k}) &\in AMIUMS \text{ and } (n+1)a_{n,n} = O(1) \end{aligned}$$

is valid, then

$$\|f - \tau_n(f)\|_{L^{p,\alpha}(\mathbb{T})} = O\left(n^{-\gamma}\right).$$

Theorem 2 *Let* $\Gamma \in \mathbb{D}$, $f \in Lip_{\alpha,p}(\mathbb{G},\gamma)$, $0 < \alpha \leq 2$, $0 < \gamma < 1$, $1 < p < \infty$ *and* $A = (a_{n,k})$ *be a lower triangular matrix with* $\left|S_n^{(A)} - 1\right| = O\left(n^{-\gamma}\right)$. *If one of the following conditions*

$$\begin{aligned} (i)\ \ (a_{n,k}) &\in AMDUMS \\ (ii)\ \ (a_{n,k}) &\in AMIUMS \text{ and } (n+1)a_{n,n} = O(1) \end{aligned}$$

is valid, then

$$\left\|f - \tau_{\mathbb{G},n}^{(A)}(f)\right\|_{L^{p,\alpha}(\Gamma)} = O\left(n^{-\gamma}\right).$$

***K*eywords** Morrey space · Morrey-Smirnov class · Matrix transforms · Faber series · Approximation

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PSEUSO-PROJECTIVE AND QUASI CONFORMAL CURVATURE TENSORS ON RIEMANNIAN SUBMERSIONS

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ABSTRACT

In this study,pseuso-projective and quasi-conformal curvature tensors on a Riemannian Submersion have been examined and on a Riemannian Submersion new curvature relations for pseuso-projective curvature and quasi-conformal curvature tensor under certain conditions have been obtained.

***K*eywords** Riemannian submersion · pseudo-projective curvature tensor · projectşvelt flat Riemannian submersion

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ENCRYPTION METHOD USING THE MOORE-PENROSE GENERALIZED INVERSE OF THE FIBONACCI MATRIX

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ABSTRACT

In this paper we introduce and study a new method in cryptology. In this method we get encryption using the Moore-Penrose generalized inverse of a square or rectangular Fibonacci matrix. We give illustrative examples about this encryption method. The singular value decomposition (SVD) of a matrix has a very important role in computation to the Moore-Penrose generalized inverse of a rectangular matrix. In this method, we use the square or rectangular Fibonacci matrix which has the Moore-Penrose generalized inverse as a key matrix. This encryption method is different from classical algebraic coding. Since this method is based on matrix multiplication, it can be performed easily and quickly by today's computer.

***K*eywords** Fibonacci Matrix · The Moore-Penrose Generalized Inverse · Encryption · Singular Value Decomposition (SVD) · Cryptology

Acknowledgements This work is supported by the Scientific Research Project (BAP) 2020FEBE003, Pamukkale University, Denizli, Turkey

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ON CONSTRUCTING BIHARMONIC MAPS ON WARPED PRODUCT MANIFOLDS

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ABSTRACT

The harmonic and biharmonic maps are correspondences between the riemannian or pseudo-riemannian manifolds which are solutions of the Euler-Lagrange equations. Let $\phi : (M^m, g) \longrightarrow (N^n, h)$ be a smooth map between Riemannian manifolds. Then ϕ is said to be harmonic if it is a critical point of the energy functional :

$$E(\phi) = \frac{1}{2}\int_M |d\phi|^2 dv_g$$

with respect to compactly supported variations. Equivalently, ϕ is harmonic if it satisfies the associated Euler-Lagrange equations :

$$\tau(\phi) = Tr_g \nabla d\phi = 0,$$

$\tau(\phi)$ is called the tension field of ϕ, one can refer to [2] for background on harmonic maps. As the generalizations of harmonic maps, biharmonic maps are defined as follows, the map ϕ is said to be biharmonic if it is a critical point of the bi-energy functional :

$$E_2(\phi) = \frac{1}{2}\int_M |\tau(\phi)|^2 dv_g$$

Equivalently, ϕ is biharmonic if it satisfies the associated Euler-Lagrange equations :

$$\tau_2(\phi) = -Tr_g \left(\nabla^\phi\right)^2 \tau(\phi) - Tr_g R^N(\tau(\phi), d\phi)d\phi = 0,$$

where ∇^ϕ is the connection in the pull-back bundle $\phi^{-1}(TN)$ and, if $(e_i)_{1\leq i\leq m}$ is a local orthonormal frame field on M, then

$$Tr_g \left(\nabla^\phi\right)^2 \tau\left(\phi\right) = \left(\nabla^\phi_{e_i}\nabla^\phi_{e_i} - \nabla^\phi_{\nabla^M_{e_i}e_i}\right)\tau\left(\phi\right),$$

where we sum over repeated indices. We will call the operator $\tau_2(\phi)$, the bi-tension field of the map ϕ. We can refer to [3], for a survey of biharmonic maps.

Clearly any harmonic map is biharmonic, therefore it is interesting to construct non-harmonic biharmonic maps (see [3] and [4] for some constructions of non-harmonic biharmonic maps). In this presentation, we will give a new class of biharmonic maps between the manifolds which admit a riemannian warped product structure (see [1]). In the first, we characterize the biharmonicity of the map $\phi : (M^m \times_\alpha N^n, G_\alpha) \longrightarrow (M^m \times_\beta N^n, G_\beta)$ defined by $\phi(x, y) = (x, y)$, with this setting we obtain new examples of biharmonic non-harmonic maps. As a second result, we will study the biharmonicity of the map $\widetilde{\phi} : (M^m \times_f N^n, G_f) \longrightarrow (P_1^{p_1} \times P_2^{p_2}, G)$ defined by $\widetilde{\phi}(x, y) = (\phi(x), \psi(y))$, where $\phi : (M, g) \longrightarrow (P_1^{p_1}, k_1)$ and $\psi : (N, h) \longrightarrow (P_2^{p_2}, k_2)$ are two harmonic maps . Finally, we study the biharmonicity of some maps on the warped product where we give some special cases.

***K*eywords** harmonic maps, biharmonic maps, warped product.

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ON DISCRETE INPUT-TO-STATE STABILITY OF BALANCE LAWS SYSTEMS

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ABSTRACT

In the present work we deal with a class of spatial-varying hyperbolic balance laws system subjected to some distributed disturbances governed by:

$$\partial_t X(z,t) + A(z)\partial_z X(z,t) + \Pi(z)X(z,t) = \delta(z,t) \qquad (z,t) \in [0,1] \times [0,+\infty) \tag{3}$$

where z and t represent, respectively, the spatial and time variables, Π and A are non-constant $n \times n$ matrices, further, the matrix A is supposed to be diagonal and satisfies the hyperbolicity condition, and δ represents the vector of disturbances.
The problem 3 is supplemented by the following conditions
Initial condition

$$X(z,0) = X_0(z) \qquad x \in (0,1) \tag{4a}$$

And boundary conditions

$$X(1,t) = K.X(0,t) \qquad \forall t \in \mathbb{R}^+, \qquad K \in \mathbb{R}^{n\times n} \tag{4b}$$

The well-posedness of such problem is discussed in different existing literatures with respect to some regularity assumptions which are assumed to be hold in our study.
The aim of the presentation is to establish input-to-state stability (ISS) properties for the considered systems class with respect to distributed disturbances based on Lyapunov functional method in both theoretical and numerical senses.

At first, we present the adopted notion of input-to-state stability as an exponential stability in the presence of disturbances. Then, by means of an explicit ISS-Lyapunov function, we will state theoretical stability results in L^2-norm.
Secondly, we migrate to the analysis of stability in the discrete sense. First, we discuss the discretization and solvability of system 3 using operator-splitting method (also called Fractional step method), the method , presented in [4], consists of splitting the non-homogeneous problem into two sub-problems: a homogeneous conservation laws system which will be discretized using upwind scheme, and simple

ODE system solved by Euler method. Then, we will define the ISS property as well as the Lyapunov function in the discrete sense. And, finally, we will be able to state the numerical stability results under appropriate assumptions.
We close by presenting an illustrative example in the purpose of testing computationally the obtained results, and some concluding remarks.

***K*eywords** Hyperbolic PDE · Balance laws systems · Input-to-state stability · Lyapunov function · In-domain disturbances · Operator-Splitting methods.

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ON WOVENNESS OF K-FUSION FRAMES

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ABSTRACT

The notion of Hilbert frames was first introduced by Duffin and Schaeffer [8] in 1952. After several decades, in 1986, frame theory was popularized by the groundbreaking work by Daubechies, Grossman and Meyer [6] by showing its practical significance in distributed signal processing. Since then frame theory has been widely applicable by mathematicians and engineers in various fields.

Frame theory literature became familiarized through several generalizations, one such generalization is K-fusion frame, K-fusion frame was first studied by Liu et al. [13]. After that Neyshaburi et al. [14] and Bhandari et al. [2] produced several characterizations of K-fusion frame.

In a sensor networking system, a frame provides a stable reconstruction of signals. If there are two frames, having similar characteristics, then absence of one or more frame elements from one of the frames still may lead to a perfect reconstruction on account of substitution by the corresponding frame elements by the other frame. In this context, basically one can think of the intertwinedness between two sets of sensors, or in general between two frames, which leads to the idea of weaving frames. Weaving frames or woven frames were first introduced by Bemrose et al. [1]. Later the concept of wovenness is characterized by Deepshikha et al. [7], Garg et al. [10] studied weaving K-fusion frames.

K-fusion frames come naturally, having significant applications, when one needs to reconstruct functions (signals) from a large data in the range of a bounded linear operator. This article presents characterizations of weaving K-fusion frames. Paley-Wiener type perturbations and conditions on erasure of frame components are discussed to examine wovenness.

***K*eywords** Frame · K-fusion frame · Weaving

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UNDERSTANDING THE LEVEL OF MATHEMATICS KNOWLEDGE OF STUDENTS WHO JOINED ISEC

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ABSTRACT

The insufficient mathematics knowledge that students have upon reaching higher education is aggravated by the heterogeneity of training of students who access engineering degrees. This multiplicity originates asymmetries in basic mathematical knowledge and difficulties higher education integration, which implies the definition of alternative paths for these students in their learning process that could help them achieve success.

Assuming that students are not learning what they should learn, because of the enormous gap in the basic knowledge, we should construct pedagogical tools that can contribute to the diagnosis, acquisition and consolidation of mathematical knowledge and skills needed in engineering, as well as develop resources that will give engineering students the best possible learning experience.

The low pass rate in the curricular units (CU) of differential and integral calculus (DIC) taught throughout the first semester of the first year of engineering degrees at the Instituto Superior de Engenharia of Coimbra led to the development of a diagnostic test with the intention of identifying the degree of knowledge, at the level of the Mathematics syllabus, considered essential for the full integration of students in the UC-CDI [1].

This diagnostic test was constructed considering the guidelines of SEFI, Mathematics for the European Engineer – A Curriculum for the Twenty-First Century -Core Zero [2]. Regarding the minimum knowledge recommended for entering higher education for an engineering course, these are detailed by areas and identified by topics in the Core Zero section. Among the suggested areas and according to the Basic and Secondary Education program of Portugal, the 20 questions were grouped taking into account the topics Algebra, Analysis and Calculus, Geometry and Trigonometry.

The analysis of the data gathered at the beginning of the first semester in the academic years 2014/15 to 2019/20 and involving students from various engineering degrees allows us to conclude the level of knowledge in terms of Mathematics content of the students who joined ISEC, as well as to realize in which topics they show the greatest difficulties. The results will lead to the development of an individual work plan, which could constitute the document of excellence for monitoring the evolution of student learning in overcoming the difficulties detected."

***K*eywords** Mathematics Knowledge · Diagnosis Test · Engineering · Differential · Integral Calculus

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A Solution of Fractional Bio-Chemical Reaction Model by Adomian Decomposition Method

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Abstract

This paper focuses on the modeling of the bio-chemical reaction viz anaerobic digestion which is biochemical process of producing biogas which is the biological degradation of biomass. This chemical phenomenon forms as an system of fractional differential equations. Therefore, the attempt has been made to model this bio-medical process and to find its solution by using powerful Adomian decomposition method. For this Caputo fractional operator is used to represent the fractional derivative.

***K*eywords** Anaerobic, · System of fractional equations, · Caputo fractional derivative.

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TOPOLOGICAL RINGS AND ANNIHILATOR CONDITIONS

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ABSTRACT

In this study, we introduce the class of rings which is defined by some annihilator conditions on projection invariant ideals. We obtain connections between the former class of rings and the class of dual rings. Examples are delimit our results.

***K*eywords** annihilator conditions · dual rings · projection invariant ideals

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FRACTIONAL REGULARITY TO THE FRACTIONAL HEAT EQUATION AND APPLICATION TO KPZ FRACTIONAL PROBLEM

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ABSTRACT

Our goal is to study a non local version of the Kardar-Parisi-Zhang equation with fractional gradient. More precisely, we consider the nonlocal problem

$$(P)\begin{cases} u_t + (-\Delta)^s u = |(-\Delta)^{\frac{s}{2}} u|^q + \mathrm{f} & \text{in} \quad \Omega_T = \Omega \times (0,T), \\ u(x,t) = 0 & \text{in} \quad (\mathbb{R}^{\mathbb{N}} \setminus \Omega) \times (0,T), \\ u(x,0) = u_0(x) & \forall x \in \Omega, \end{cases}$$

where Ω is a bounded domain of $\mathbb{R}^N$ for $N > 2s$ and $0 < s < 1$. We assume that f and u_0 are non negative functions. The above problem is related to the fractional Laplacian defined by

$$(-\Delta)^s u(x,t) := a_{N,s} PV \int_{\mathbb{R}^N} \frac{u(x,t) - u(y,t)}{|x-y|^{N+2s}} dy.$$

It is also defined as the multiplier of the Fourier transform with symbol $||\xi||^{2s}$. Namely for $\phi \in S(\mathbb{R}^{\mathbb{N}})$, we have

$$\mathcal{F}((-\Delta)^s \phi)(\xi) = |\xi|^{2s} \mathcal{F}(\phi)(\xi).$$

The aim of our work is to show the existence and the fractional regularity of solution to problem (P) according to the value of q and the regularity of the data f and u_0.

Our existence result holds also for different variant of the nonlocal gradient term. Namely the term can be substituted by $|\nabla^s u(x,t)|^q$ where

$$\nabla^s u(x,t) := \int_{\mathbb{R}^{\mathbb{N}}} \frac{u(x,t) - u(y,t)}{|x-y|^s} \frac{x-y}{|x-y|} \frac{dy}{|x-y|^N}, \quad \forall\, x \in \mathbb{R}^N. \tag{5}$$

or

$$\mathbb{D}_s(u)(x,t) = \left(\frac{a_{N,s}}{2} \int_{\mathbb{R}^N} \frac{|u(x,t) - u(y,t)|^2}{|x-y|^{N+2s}} dy \right)^{\frac{1}{2}}. \tag{6}$$

which is related to the Bessel potential space.

In order to prove the existence result for the problem (P) we need to analyze deeply the question of fractional regularity to the corresponding heat fractional equation with Dirichlet condition. More precisely we prove the next global regularity result.

Theorem 1 Let u be the unique weak solution to the problem

$$\begin{cases} u_t + (-\Delta)^s u = \mathrm{f} & \text{in} \quad \Omega_T = \Omega \times (0,T), \\ u(x,t) = 0 & \text{in} \quad (\mathbb{R}^{\mathbb{N}} \setminus \Omega) \times (0,T), \\ u(x,0) = 0 & \forall x \in \Omega, \end{cases} \tag{7}$$

with $f \in L^m(\Omega_T)$, then $(-\Delta)^{\frac{s}{2}} u \in L^q(\mathbb{R}^{\mathbb{N}})$ for all $q < \frac{m(N+2s)}{(N+2s-ms)_+}$. Moreover $u \in L^q(0,T,L_0^{q,s}(\Omega))$, the corresponding parabolic Bessel potential space, for all $q < \frac{m(N+2s)}{N+2s-ms}$.

The proof of the previous regularity result is achieved using the representation formula and a fine estimate on the fractional gradient of the heat kernel $P_\Omega(x,y,t)$. Recall that P satisfies

$$P_\Omega(x,y,t) \backsimeq \left(1 \wedge \frac{\delta^s(x)}{\sqrt{t}}\right) \times \left(1 \wedge \frac{\delta^s(y)}{\sqrt{t}}\right) \times \left(t^{\frac{-N}{2s}} \wedge \frac{t}{|x-y|^{N+2s}}\right). \tag{8}$$

Once proving the corresponding estimate on P_Ω, we get the next existence result.

Theorem 2: Suppose that $f \in L^m(\Omega_T)$ with $m > \frac{(N+2s)}{q's}$ and $u_0 = 0$, then (P) has a weak solution u such that $u \in L^\sigma((0,T;L_0^{s,\sigma}(\Omega))$ for all $\sigma < \frac{m(N+2s)}{N+2s-ms}$.

The above relation between q and m is optimal in order to get a solution to problem (P).

Keywords fractional Laplacian · fractional gradient · Bessel potentiel space · fractional regularity

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ARITHMETIC REPRESENTATION OF BOOLEAN VALUED FUNCTIONS IN THE BINARY DYNAMICAL SYSTEMS

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ABSTRACT

The aim of this work is to determine the relationship between Boolean valued discrete vector functions and a functional that characterizes the optimal processes in the binary dynamical systems[1,2]. For this reason, system model is considered as in nonlinear and lineer case. Also, in order to solve the problem given in that systems, a mathematical approach is improved between these two different types of functions[3,4,5].

***K*eywords** Arithmetic representation · Booelan function · binary dynamical system

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TEAM-BASED LEARNING COLLABORATIVE, IS POSSIBLE ONLINE?

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ABSTRACT

Collaboration is the process of two or more people, entities or organizations working together to complete a task or achieve a goal. Especially in this digital era where "collaboration ware" is rapidly growing and greatly facilitates the communication and exchange of thoughts and information without restrictions of time and place, "online collaboration" has become an effective means of learning. Team-based learning collaborative (TBL-C) is a structured form of small-group learning that emphasizes student preparation out of class and application of knowledge in class. Students are organized strategically into diverse teams of 5-7 students that work together throughout the class. Before each unit or module of the course, students prepare by reading prior to class.
With the Covid-19 pandemic, teaching/learning at universities like Coimbra Institute of Engineering undergoes the transition to online teaching and the TBL-C approach also had to be adjusted to an online learning environment.
This paper presents the using the TBL-C in a Miro platform to teach mathematic for first year electrotechnical engineers. The numerical integration (trapezoidal rule and Simpson's rule) content was acquired by students in a motivation and effective form using Miro platform with TBL-C methodology. During the classes, the following positive points were identified: promotion of student learning, readiness to perform tasks, timely feedback on student progress and guidelines that should be advised to students, and finally the most important, guarantee of success from the students. In conclusion, the TBL-C methodology are more interesting, enjoyable, and motivational than traditional methodologies and allows student success.

***K*eywords** Team-based learning · Collaborative learning · Remote learning

GEOGEBRA AUGMENTED REALITY: IDEAS FOR TEACHING & LEARNING MATH

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ABSTRACT

To attract students and unleash opportunities for teaching and learning mathematics, opening the minds of students and teachers to creative thinking, Geogebra Augmented Reality (AR) has been adopted in education for some authors. In relation to higher education and its application in mathematics, there are also some recent studies. Within this context, a new teaching strategy for Integral Calculus syllabus was developed, where learning with technologies and technologies in learning were used.

In the academic year 2020/2021, and in continuation with the work already done previously, it was proposed to students of Calculus 1 course in Electrotechnical Engineering at Coimbra Institute of Engineering, the construction of a real object 3D in Geogebra that can be considered a solid of revolution and with AR technology to visualize it in real context, being able to explore the solid at different angles as the device's camera moves freely. Questionnaires were made to the students as a way to obtain the interest and satisfaction of the student for this type of activities as well as the students' performances in these contents were analysed in written exams to confirm if the learning was carried out successfully.

GeoGebra AR can and should be used to complement the teaching and learning of mathematics for engineering students. When students are involved with Geogebra AR, they expand their range of educational tasks, including STEM (Science, Technology, Engineering and Mathematics) problems. This makes it possible to achieve a high level of learning motivation and to individualize the learning process. The concepts of application of integral calculus are explored when the student uses the tools. The learning process becomes extremely pleasurable, which gives the student a significant knowledge of both the mathematical concepts and the geometry involved.

***K*eywords** Math teaching · Augmented Reality · Engineering students.

FROM FIBONACCI SEQUENCE TO MORE RECENT GENERALISATIONS

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ABSTRACT

Number sequences have been the subject of several research studies. From the algebraic properties to the generating matrices and generating functions of these sequences, all these topics and many others have been studied and a vast bibliography covers this type of sequences. The Fibonacci succession has been one of the genesis of multiple researchs that resulted in the creation of other number sequences and respective research. Many papers are dedicated to Fibonacci and Lucas sequences, such as the works of Dil and Mezo, in [1], Hoggatt, in [2], Vorobiov, in [3], among so many others. In this context, based on the most recent works on this subject, we develop new generalisations of these recurrence successions, introducing the definition, properties and some theorems concerning the k-Fibonacci and k-Lucas numbers, as well as k-Jacobsthal and k-Jacobsthal-Lucas [4], not forgetting also the k-Pell, k-Pell Lucas and Modified k-Pell numbers [5], as well as Balancing and Cobalancing numbers [6] and some their generalisations. In the meanwhile, more recently, the definition and properties of the Incomplete numbers [7], the k-Telephone numbers [8] and the Leonardo numbers [9, 10, 11] have resulted in new papers already published or already submitted. The introduction of the defintion of Hybrid number [12] has been the aim of so many researchers to study the Hybrid version of several numbers sequences. For instance, we can find in the literature the studies of Szynal-Liana and Włoch [13, 14], Kızılates in [15], Szynal-Liana in [16], Catarino and Bilgici in [17], Catarino in [18], and so many others. For each of these sequences we found, for instance, the formulas of Binet, Catalan, Cassini and d'Ocagne, and we analysed the convergence of the quotients of consecutive terms of these successions, checking whether, analogously to the Fibonacci succession,

they are close to some metallic number. Also the Hyper k-Pell, Hyper k-Pell-Lucas and Hyper Modified k-Pell [19] sequences are introduced, as well as their generating functions and some properties of each one. Furthermore, properties about the relationship between them and their generating functions are presented. In addtion, the convexity, concavity, log-concavity and log-convexity properties for these sequences are established.

***K*eywords** k-Fibonacci numbers · k-Jacobsthal numbers · k-Pell numbers · Balancing numbers · Incomplete numbers · k-Telephone · Hyper k-Pell numbers.

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SHEPARD OPERATOR OF LEAST SQUARES THIN-PLATE SPLINE TYPE

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ABSTRACT

One of the best suited methods for approximating large sets of data is the Shepard method, introduced in [4]. It has the advantages of a small storage requirement and an easy generalization to additional independent variables, but it suffers from no good reproduction quality, low accuracy and a high computational cost relative to some alternative methods [3], these being the reasons for finding new methods that improve it.

In the paper [1] we have obtain some new Shepard type operators based on the classical, the modified Shepard methods and the least squares thin plate spline function. Given some sets of points, we compute some representative subsets of knot points following an algorithm described by J. R. McMahon in [2].

***K*eywords** Scattered data · Shepard operator · least squares approximation · thin plate spline · knot points.

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SOME CASES OF RELAXED ELASTIC LINES IN 3-DIMENSIONAL QUASI-SASAKIAN MANIFOLDS

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ABSTRACT

We find the classification of the curvatures of relaxed elastic lines in 3-dimensional quasi-Sasakian manifolds with C-parallel and C-proper mean curvature vector field in the tangent and normal bundles. Moreover, we study slant curves satisfying relaxed elastic line conditions in 3-dimensional quasi-Sasakian manifolds. We also give examples of such lines.

***K*eywords** relaxed elastic line · slant curve · C-proper mean curvature vector field

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JOINTLY TYPE-II CENSORED LENGTH-BIASED EXPONENTIAL DISTRIBUTIONS

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ABSTRACT

In the reliability theory, data censoring scheme is a very important topic and handled by various researchers. It is known that the lifetimes of components or units may not be always recorded exactly. In most cases, components or units are lost or removed from the experiments before they fail. In these cases, the censored datasets are observed. On the other hand, the conventional censoring schemes are mostly deal with one sample problem in the reliability theory. However, when the experimenters decide to conduct comparative life tests of units, joint censoring schemes are needed. For this purpose, joint censoring schemes have been suggested in the literature.

Type-II censoring plan is a mostly used censoring scheme in life test experiments. In this censoring scheme, r units in a random sample of size n $(r < n)$ are observed and the test terminates with m-th fail. The jointly type-II censoring schemes is constructed similarly. We first suppose that two independently and identically distributed random variables (X, Y) follow specific probability distributions with sample sizes n and m where $N = n + m$, respectively. Then, it is assumed that $W_{(1)}, W_{(2)}, \cdots, W_{(N)}$ denote the combined order form of test samples $\left\{X_{(1)}, X_{(2)}, \cdots, X_{(n)}; Y_{(1)}, Y_{(2)}, \cdots, Y_{(m)}\right\}$. Such a test terminates with the $r-$th pre-specified failures, $W_{(r)}$.

On the other hand, Dara and Ahmad (2012) introduced the length-biased exponential (LBE) distribution and they proved that the LBE distribution is more flexible than the exponential distribution. Unlike the exponential distribution has constant failure rate, the LBE distribution is an increasing failure rate (IFR) class of distribution. It is known that an IFR component has the better chance of surviving any shorter period and the worse chance of surviving any longer period.

This paper deals with the jointly type-II censored length-biased exponential populations. We first consider the maximum likelihood estimators of the unknown rate parameters along with their asymptotic confidence intervals. Then, the Bayesian

inference procedure is provided with the highest posterior density credible intervals. In Bayesian estimations, importance sampling methods is used. The whole theoretical studies are illustrated with simulation and real data studies.

***K*eywords** Bayesian inference · Jointly censoring · Length-biased exponential distribution · Maximum likelihood estimation

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ON PARABOLIC AND ELLIPTIC ELEMENTS OF MODULAR GROUP

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ABSTRACT

Modular group $\Gamma = PSL(2,\mathbb{Z})$ is the projective special linear group of $2x2$ matrices over the ring of integers with determinant 1. Modular group acts on the upper half of the complex plane $\hat{\mathbb{H}}$ via linear fractional transformations. These transformations are orientation preserving isometries of $\hat{\mathbb{H}}$. Γ is generated by;

$$x = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad u = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Let $y = x.u \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$ then Γ is isomorphic to free product of two cyclic groups of orders 2 and 3. Hence Γ has presentation;

$$\Gamma =< x, y : x^2 = y^3 = 1 > \approx \mathbb{Z}_2 * \mathbb{Z}_3$$

Using the presentation of Γ every element can be expressed as a word of x and y or x and u. There are tree types of elements in Γ. Let $V = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma$ then V is hyperbolic if $|tr(V)| > 2$, parabolic if $|tr(V)| = 2$ and elliptic if $|tr(V)| < 2$. An element in Γ is finite ordered if and only if it is elliptic. Thus elliptic elements are used in the presentation of Γ.

In this talk we obtain a criteria for existence of parabolic and elliptic elements in Γ with given cusp point $\frac{p}{q} \in \mathbb{Q}$. Cusp point is basically the image of ∞ under the coresponding transformation of an element. After we construct these elements as a word in generators using continued fractions and Farey graph.

Keywords Modular group · Continued fractions · Farey graph

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EXTENSION OF LEAP CONDITION IN APPROXIMATE STOCHASTIC SIMULATION ALGORITHMS OF BIOLOGICAL NETWORKS WITH 2^{nd} AND 3^{rd} ORDER TAYLOR EXPANSION

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ABSTRACT

Stochastic simulation algorithms (SSAs) is an exact Monte Carlo procedure to simulate numerically time trajectories of molecular populations in the biological systems. The three main approaches of SSAs are the direct method (Gillespie algorithm), the first reaction method and the next reaction method. The SSA gives the exact result but then it takes slowly in the large systems because of trying to calculate every different reaction event. In the view of computationally demanding, it can be used approximate SSAs. These methods depend on the leap condition. This implies that there should be no significance change in the propensity function during the time interval t to $[t, t+\tau]$. Here, the study aims to construct an actual and close confidence interval for the parameter denoting the number of simultaneously reaction in the system, by advancing the leap condition and expanding the hazard function by second and third order Taylor expansion in the time interval $[t, t+\tau]$. To reach the goal, we use the poisson τ-leap, which is one of the fundamental approximate SSA in the literature, and approximate Gillespie algorithm. Moreover, we derive the maximum likelihood estimators (MLE) and the method of moment estimators (MME) of the simulation parameters and construct confidence interval estimators at a given significance level α for these algorithms. Finally, from these derivations, we theoretically present that our new confidence intervals are narrower than the current confidence interval in the leap condition to get more potential values of parameters. [1], [2], [3], [4], [5], [6], [7].

***K*eywords** Approximate stochastic simulation algorithms · Leap condition · Confidence interval · Taylor expansion

Acknowledgement: The authors thank to the Scientific Research Project of Middle East Technical University (Project no: BAP- 10282) for their suppport.

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FORECASTING OF COVID 19 USING DEEP LEARNING METHODS

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ABSTRACT

Modeling the daily number of cases and the number of deaths in Covid 19 is difficult due to its stochastic structure and the presence of variables affecting the event. It is not possible to make predictions and find the best prediction, especially for time-dependent data. Because different models give the best results according to different criteria. A lot of work has been done to model Covid 19, which is today's epidemic disease. and many of these studies were conducted with monthly, quarterly, or semi-annual data. The difference of this study from the others is that it covers long-term data and the effect of vaccination is included in the model. In this study; In order to analyze the daily active cases, recovery and death trends in Turkey, the long short-term memory (LSTM) deep learning method was compared with the time series method. For this purpose, Turkey's daily case and death numbers between March 11, 2020 and June 1, 2021 were used and all models were coded in the R software programming language. Obtained results were compared with MAE, RMSE, MPE and MAPE criteria. As a result of the analysis, it was observed that the LSTM method gave better results than the time series.Modeling the number of cases and deaths in Covid-19 in the most realistic way allows the state to make an action plan to reduce the number of cases by making decisions at the most appropriate time.According to the results obtained, the LSTM method can be used to estimate the number of Covid 19 cases according to the comparison criteria.

***K*eywords** Covid 19 · Deep Learning · LSTM · Time Series · Forecasting

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Optimization of Discrete-approximation Problem and Equivalence Theorem of Subdifferential

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Abstract

In this paper, we formulate necessary and sufficient conditions for the optimality of the underlying problem using convex and nonsmooth analysis and non-degeneracy assumptions imposed on the optimal trajectory for a convex problem. A system of second-order differential inequalities involving first- and second-order derivatives of searched functions is used to provide system dynamics in this paper's optimal control discrete-approximation problem. Differential inequalities occur naturally in a wide range of applied mathematics areas in science and engineering, such as mathematical economics in resource allocation problems and the study of planning procedures, mechanics, and differential games. Particular equivalence theorems are needed to pass to the discrete approximation problem, which in turn links the main results of systems of second-order discrete and discrete approximation problems. We use first and second-order difference operators in the discrete approximation problem associated with the continuous problem. Moreover, Equivalence theorems, which are similar results for the non-convex problem have been demonstrated using local tents and CUAs. Equivalence theorems, which are effective tools for studying discrete approximation problems, can also be applied in numerical methods.

***K*eywords** Approximation · Equivalence Theorem · Inclusion

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THE PERIODICITY OF THE DETERMINANT OF A $(0, 1)$ DOUBLE BANDED MATRIX

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ABSTRACT

Let $\mathbb{H}_n$ be the $(2n+1)$-dimensional Heisenberg group. In this paper, we give some group invariant properties of the Factorized Heisenberg Laplacian.

EXPLICIT FORMULA FOR THE n-TH DERIVATIVE OF A QUOTIENT

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ABSTRACT

Leibniz's rule for the n-th derivative of a product is a very well known and extremely useful formula. In this article, we introduce an analogous explicit formula for the n-th derivative of a quotient of two functions. Later, we use this formula to derive new partition identities and to develop expressions for some special n-th derivatives.

***K*eywords** n-th derivative of a quotient · generalized quotient rule · partitions.

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SEMIGROUP AND MONOID CONSTRUCTION OF SOME POLYGONAL NUMBERS

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ABSTRACT

Polygonal numbers are natural numbers that can be represented by regular geometric shapes. Polygonal numbers start from certain point and continue to increase by the same common difference. If the common difference is one, then the geometric structure is called a triangular number. If it is two, then it becomes a square; if it is three, then it becomes a pentagonal number. And so on.

Many special numbers have been created by taking inspiration from polygonal numbers. Pythagoras triples, Perfect numbers, Mersenne numbers, Cullen Numbers, Woodal Numbers, Fermat numbers, Lucas numbers,Thabit numbers, etc. are such numbers.

Binary operators have played an important role in many algebraic structure. Such as, groupoid, semigroup and monoid. In this direction there are given some binary operators of groupoids in [2,3].

In this talk, by using similar methods we give some binary operators on the polygonal numbers and investigated whether they are semigroup and monoid or not.

***K*eywords** Polygonal Numbers · Binary Operator · Semigroup and Monoid

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THE PERIODICITY OF THE DETERMINANT OF A $(0,1)$ DOUBLE BANDED MATRIX

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ABSTRACT

In this talk, we establish the periodicity of the determinant of a $(0,1)$ double banded matrix. As a corollary, we answer to two recent conjectures and other extensions. Several illustrative examples are provided as well. This is a result of jointly publications with Milica Anđelić (Kuwait University, Kuwait) and Zhibin Du (South China Normal University, Foshan, Guangdong, China).

***K*eywords** Determinant · double banded matrices · pentadiagonal matrices

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TRAVELLING WAVES IN GAS FLOW THROUGH POROUS MEDIA WITH ADSORPTION

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ABSTRACT

The issue of excessive amounts of carbon in the atmosphere, which is still being added to at an alarming rate, and the resultant effect on the climate is well-documented. Consequently, mankind must look to a range of solutions including a reduction in current production, removing existing carbon from the environment and safe storage or reuse. The process of carbon capture falls within these measures. In this talk we will present a mathematical model for the flow of a CO_2 laden gas through an adsorbing filter. The model consists of an advection-diffusion equation for the gas concentration, coupled to a momentum equation for the gas velocity and an ordinary differential equation for the rate of adsorption. By nondimensionalizing the model, we will retain the most dominant terms and neglect the less important ones. We will show how travelling wave solutions, asymptotic analysis and numerical methods can all be applied to gain understanding on the industrial process of contaminant removal by adsorption.

***K*eywords** Travelling waves · Porous media · Mass transfer

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SEMI-LATTICES AND CONGRUENCES IN ROUGH NEUTROSOPHIC SET MODEL

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ABSTRACT

In this study, we address rough neutrosophic sets from the set-theoretic point of view. In what follows, we give an analysis of abstract mathematical context on the notion of independence and then formulated in the language of semi-lattices and congruence. We consider a semi-lattice along with an equivalence relations with respect to semi-lattice operation. The final part of this study is the presentation of the notion of a dependence which will be an abstract counterpart of the notion of functional dependence.

***K*eywords** Dependence space · Reducts · Approximation sets

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TEACHING MULTIPLE INTEGRALS TO ENGINEERS: ESTIMATING THE REAL VOLUME OF A BUILDING EXCAVATION, FROM A POINT CLOUD COLLECTED BY TOPOGRAPHY MEASUREMENTS

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ABSTRACT

Teaching calculus in several variables to Engineers is complicated. Our stu-dents generally have some problems understanding the notation, and they do not like the theoretical part of our subjects, additionally many of our subjects are in first course, and therefore they do not understand the usefulness of the tools that we are providing them. Thus, we considered a good idea to invite a student in third or fourth course to explain why calculus in multiple variables is important to solve real life problems, giving examples of applications that he found in other types of subjects (not mathematics). In this work, we solve one of them taken in a specific and real topographic problem: The activity that we developed is compatible with e-learning methodologies. This consists in estimating the real volume of excavated material needed to raise an industrial building, from a series of measurements made with a theodolite on a plot of a nearby town. With the help of Mathematical software and interpolation methods, students can compare their results with others obtained with other commercial programs available for Engineers. In this way, they have to use different mathematical tools to develop their mathematical skills through competencies, and they understand that their subjects, or topics in a subject are not watertight compartments.

***K*eywords** Multiple integrals · interdisciplinary activities · Engineering and science students

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ECONOMIC GROWTH MODELS WITH OPTIMAL CONTROL PROBLEMS AND THEORY OF LIE GROUPS

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ABSTRACT

This work deals with the analysis of Ramsey dynamical model with current Hamiltonian defining an optimal control problem in a neoclassical growth model via Lie group theory. Lie point symmetries of coupled nonlinear first-order ordinary differential equations corresponding to first-order conditions of maximum principle are investigated and then first integrals and corresponding closed-form (analytical) solutions are obtained by using Lie point symmetries in conjunction with Prelle-Singer and Jacobi last multiplier methods. Besides, associated λ-symmetries, adjoint symmetries, Darboux polynomials of the Ramsey model are presented.

***K*eywords** Ramsey dynamical model, Lie point symmetries, Prelle-Singer approach, Jacobi last multiplier, closed-form solution.

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AN EXAMPLE OF MIXED BROWNIAN MOTION

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ABSTRACT

In this paper, firstly, we introduce a new gaussian process as an extension of the well known bifractional Brownian motion as a linear combination of a finite number of independent bifractional Brownian motions. We have chosen to call this process the mixed bifractional Brownian motion. Secondly, we study some stochastic properties and characteristics of this process: The Holder continuity, the self similarity, the quadratic variation, the Markov property and the differentiability of the trajectories, the long-range dependence, the stationarity of the increments and the behavior of the noise generated by the increments of this process. We believe that our process can be a possible candidate for models which involve self similarity, long range dependence and non-stationarity of

***K*eywords** Gaussian process · Self similarity · Brownian motion · Bifractional Brownian motion · Quadratic variation · Differentiability · Long range dependence.

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THE GLOBAL ASYMPTOTIC STABILITY OF A P-DIMENSIONAL DIFFERENCE EQUATIONS SYSTEM

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ABSTRACT

The global asymptotic stability of the unique positive equilibrium point and the rate of convergence of positive solutions of the system of two recursive sequences has been studied recently. Here we generalize this study to the system of p recursive sequences $x_{n+1}^{(j)} = A + (x_{n-m}^{(j+1)mod(p)}/x_n^{(j+1)mod(p)})$, $n = 0, 1, \ldots, m, p \in \mathbb{N}$, where $A \in (0, +\infty)$, $x_{-i}^{(j)}$ are arbitrary positive numbers for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, p$. We also give some numerical examples to demonstrate the effectiveness of the results obtained.

***K*eywords** System of fractional difference equations · periodic solutions · Boundedness · the asymptotic behavior · Stability.

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CONTENT BASED IMAGE RETRIEVAL USING HDMR CONSTANT TERM BASED CLUSTERING

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ABSTRACT

The studies related with the content-based image retrieval (CBIR) has increased because of both necessity for efficient image retrieval and the limitations in large-scale systems. Efficient image retrieval refers to finding accurate image from the database with high speed. This paper presents a new efficient image retrieval method using High Dimensional Model Representation (HDMR). The method has two main steps, clustering and retrieval. In clustering part, we use k-means method on HDMR constant term while in the subsequent part, we retrieve the most similar images to a given query image from a relevant cluster. We experiment the efficiency and effectiveness of the new algorithm on Columbia Object Image Library (COIL100) and get conspicuous results. These results are tabulated in the paper.

***K*eywords** HDMR · Image retrieval · Image decomposition · Clustering · k-means

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THE COMPATIBILITY OF OFFSHORE WIND DATA WITH THE WEIBULL DISTRIBUTION FUNCTION WITH USING THE MOVING LEAST SQUARES APPROXIMATION

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ABSTRACT

In this study, the fit of wind data to the weibull distribution function (WDF) has been investigated by using real data. While finding the coefficients of the Weibull distribution function, the moving least squares approximation is used. The performance of the moving least squares method was also examined on the offshore, and the results were evaluated in different statistical error analysis tests. There are many studies related to wind energy subject, the speed of the wind is taken as a random parameter. It is very important to know the distribution of wind measurement data during the long period. In this study, one-year data was examined and the results were evaluated monthly. Iskenderun Gulf was selected to examine the compatibility of offshore wind data with WDF function. Turkey is a developing country makes a lot of investment in the energy field.

***K*eywords** Weibull Distribution, Graphical Method, Moving Least Squares Method, Statistical Test

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ON A GENERALIZATION OF FI-EXTENDING MODULES.

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ABSTRACT

In this paper, we introduce modules with the property that every f-closed submodule has a complement which is a direct summand. We provide some structural properties related to the class of generalization of extending modules.

***K*eywords** complement submodule · extending module · fully invariant submodule.

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FORECASTING THE MONTHLY AIR PASSENGERS WITH RNN, GRU AND LSTM METHODS IN THE COVID-19 PERIOD

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ABSTRACT

The covid-19 pandemic has negatively affected the transportation and tourism sectors. Although the magnitude of this effect can be reduced by various measures, it is estimated that it will last for many years. Since the countries blocked the entrances and exits to prevent the spread of the virus, there have been significant decreases in the number of air passengers. As countries blocked the entrance and exit to prevent the spread of the virus, there have been very significant decreases in the number of air passengers. Turkey, which is one of the most important transit points, was particularly affected by this situation.

It is necessary to forecast the number of air passengers in order to manage the crisis correctly and to minimize the losses at the airports. For this purpose, in our study, the monthly total number of air passengers was modeled with recurrent neural network (RNN), gated recurrent unit neural network (GRU) and long short-term memory neural network (LSTM) methods. Obtained results were compared with MAE, RMSE, MPE and MAPE criteria. The results show that deep learning algorithms produce successful results in forecasting the number of monthly air passengers. As a result, we observed that the GRU method gives the best forecasting results.

***K*eywords** Covid-19 · Deep Learning · LSTM · RNN · GRU · Air Passenger Forecasting

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ON HARMONIC COMPLEX FIBONACCI SEQUENCES

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ABSTRACT

In this study, by inspiring the recent papers, we define a new type of number system called the harmonic complex Fibonacci sequences which combines harmonic and complex numbers. Then, some fundamental definitions and theorems about this system are given. Furthermore, algebraic properties such as Binet-like-formula, partial sums, generating function related to these sequences are denoted.

***K*eywords** Harmonic numbers · Complex numbers · Binet-like-formula

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ON DARBOUX SLANT RULED SURFACES GENERATED BY NATURAL LIFT CURVES

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ABSTRACT

In this study, an isomorphism between unit dual sphere, DS^2 and the subset of tangent bundle of unit 2-sphere, $T\bar{M}$ is represented. According to E. Study mapping, a ruled surface in IR^3 corresponds to each curve on DS^2. Using this isomorphism, new forms of ruled surfaces called Darboux slant ruled surfaces generated by natural lift curves in IR^3 are introduced. Furthermore, the correspondence between Darboux slant ruled surfaces and other slant ruled surfaces are given.

***K*eywords** Natural lift curve · Ruled surface · Slant ruled surface

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Δ^f-LACUNARY STATISTICAL CONVERGENCE OF ORDER β

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ABSTRACT

Let f be an unbounded modulus, $X=(X_k)$ be a sequence of fuzzy numbers, $\theta=(k_r)$ be a lacunary sequence, β be a real number such that $0<\beta\leq 1$ and Δ be a difference operator such that $\Delta X=X_k-X_{k+1}$. We say that the fuzzy sequence $X=(X_k)$ is Δ^f-lacunary statistically convergent to fuzzy number X_0 of order β if there is a fuzzy number X_0 such that

$$\lim_{r\to\infty}\frac{1}{f\left(h_r\right)^{\beta}}.f\left(\left|\left\{k\in I_r:d\left(\Delta X_k,X_0\right)\geq\varepsilon\right\}\right|\right)=0$$

where $I_r=(k_{r-1},k_r]$ and $f\left(h_r\right)^{\alpha}$ denotes the βth power of $f\left(h_r\right)$. that is $\left(f\left(h_r\right)^{\beta}\right)=\left(f\left(h_1\right)^{\beta},f\left(h_2\right)^{\beta},...,f\left(h_r\right)^{\beta},...\right)$. This space will be denoted by $S_{\theta}^{f,\beta}\left(\Delta_F\right)$. In this case, we write $S_{\theta}^{f,\beta}\left(\Delta_F\right)-\lim X_k=X_0$ or $X_k\to X_0(S_{\theta}^{f,\beta}\left(\Delta_F\right))$. On the other hand, let $p=(p_k)$ be a sequence of strictly positive real numbers. We say that the sequence $X=(X_k)$ is strongly $W_{\theta}^{f,\beta}\left[\Delta_F,p\right]-$summable to fuzzy number X_0 if there is a fuzzy number X_0 such that $\frac{1}{h_r^{\beta}}.\sum_{k\in I_r}[f\left(d\left(\Delta X_k,X_0\right)\right)]^{p_k}\to 0$, as $r\to\infty$. In the present case, we denote $W_{\theta}^{f,\beta}\left[\Delta_F,p\right]-\lim X_k=X_0$. Similarly, we say that the sequence $X=(X_k)$ is strongly $W_{\theta}^{f,\beta}(\Delta_F,p)-$summable to fuzzy number X_0 if there is a fuzzy number X_0 such that $\frac{1}{f(h_r)^{\beta}}.\sum_{k\in I_r}[f\left(d\left(\Delta X_k,X_0\right)\right)]^{p_k}\to 0$, as $r\to\infty$. Furthermore, we say that the sequence $X=(X_k)$ is strongly $W_{\theta,f}^{\beta}(\Delta_F,p)-$summable to fuzzy number X_0 if there is a fuzzy number X_0 such that $\frac{1}{f(h_r)^{\beta}}.\sum_{k\in I_r}(d\left(\Delta X_k,X_0\right))^{p_k}\to 0$, as $r\to\infty$. The main object of this article is to introduce the concepts of Δ^f-lacunary statistical convergence of order β and strong Δ^f-lacunary summability of order β for sequences of fuzzy numbers and define some seqeunce classes related to these concepts. We give some inclusion relations between those seqeunce classes. Fur-

thermore, let $\theta = (k_r)$ and $\theta' = (s_r)$ be two lacunary sequences such that $I_r \subset J_r$ for all $r \in \mathbb{N}$ and β_1, β_2 be two real numbers such that $0 < \beta_1 \leq \beta_2 \leq 1$, where $I_r = (k_{r-1}, k_r]$, $h_r = k_r - k_{r-1}$ and $J_r = (s_{r-1}, s_r]$, $\ell_r = s_r - s_{r-1}$. Then, we get interesting results such as uniqueness of limit for two lacunary sequences which have defined in different intervals as above.

***K*eywords** Statistical convergence · Fuzzy sequence · lacunary sequence · modulus function · difference operator

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FROBENIUS NUMBERS, SYLVERTER NUMBERS AND SUMS ASSOCIATED WITH NUMBER OF SOLUTIONS

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ABSTRACT

Let $a_1, a_2, \ldots, a_k$ be positive integers with $\gcd(a_1, a_2, \ldots, a_k) = 1$. Frobenius number is the largest positive integer that is NOT representable in terms of $a_1, a_2, \ldots, a_k$. When $k \geq 3$, there is no explicit formula in general, but some formulae may exist for special sequences $a_1, a_2, \ldots, a_k$, including, those forming arithmetic progressions and their modifications. We give formulae for the power and weighted sum of nonrepresentable positive integers. As applications, we show explicit expressions of these sums for $a_1, a_2, \ldots, a_k$ forming arithmetic progressions.

***K*eywords** Frobenius problem · Frobenius numbers · Sylverter numbers · Sylvester sums · power sums · weighted sums · arithmetic sequences

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ON THE GENERALIZED FIBONACCI POLYNOMIALS

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ABSTRACT

Fibonacci polynomials, a type of polynomial sequence that can be regarded as an extension of the Fibonacci numbers, were defined by Eugene Charles Catalan in 1883 (see [1]). Numerous studies on the generalizations (*k*-Fibonacci polynomials, *h(x)* generalization of Fibonacci polynomials, q-Fibonacci polynomials, (p,q)-Fibonacci polynomials etc.) and properties of these polynomials have been conducted in special function theory and number theory, some of which can be seen in [2, 3, 4, 5]. Some special polynomials such as Fibonacci polynomials can be stated in a variety of ways, including explicit formula, hypergeometric function form and integral representation. In [6], for example, Erkus-Duman and Ciftci obtained hypergeometric function form of Fibonacci polynomials. On the other hand, Cesarano et al. defined the general states of certain orthogonal polynomials, including the first and second kinds of Chebyshev and Gegenbauer polynomials, as integral representation in [7]. We use an integral representation of Fibonacci polynomials to define a new generalization of Fibonacci polynomials, which is motivated by the work [7]. As a result, the aim of this paper is to generalize Fibonacci polynomials using Hermite polynomials of the Kampé de Fériet type. Firstly, Fibonacci polynomials and generalized two-variable Fibonacci polynomials are defined as integral representations, which include Kampé de Fériet type Hermite polynomials. Then, we present a generating function, an explicit formula, hypergeometric function form and recurrence relations for the generalized two-variable Fibonacci polynomials. Some particular cases of these polynomials will also be demonstrated in detail towards the conclusion of the paper.

Keywords Fibonacci Polynomials · Kampé de Fériet type Hermite polynomials · Integral representation · Generating function · Hypergeometric function · Recurrence relation

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ON QUATERNION ALGEBRA

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ABSTRACT

Quaternions, which have been widely used since the 18th century, are a kind of generalization of complex numbers. It was introduced and defined as a new number system in 1843 by the Irish mathematician Sir William Rowan Hamilton. Thus, Hamilton included quaternions into the concept of numbers. Quaternions, which have some properties different from complex numbers and real numbers, have a real algebra structure that is not commutative. That's why; they have most of the properties of complex numbers. By describing quaternions, Hamilton introduced a new multiplication operation into vector algebra where division for two vectors is also possible. Quaternions corresponding to rotations in three-dimensional space have also facilitated the study of motions in Euclidean space. In this study; the algebraic structure of quaternions on the $(Z_p(e_1), +, .)$ field obtained by using prime numbers is constructed. A product is defined in the $IH/Z_p(e_1)$ vector space and is called the quaternion product. Thus, it has been shown that the $IH/Z_p(e_1)$ quaternion product has the properties of a quaternion algebra. In addition, with the help of a linear transformation, the matrix representation and properties of this kind of quaternion algebra, defined on the object obtained by using prime numbers, are given. The $q \to T_q$ transform is obtained where the $\{IH/Z_p(e_1), \oplus, Z_p(e_1), +, ., \odot, \times\}$ quaternion algebra is a2×2 type matrix representation defined on the $(Z_p(e_1, +, ., \odot)$ field.

***K*eywords** Ring · Field · Quaternions · Quaternion Algebra

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AN APPROACH TO SPLIT QUATERNIONS ALGEBRA

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ABSTRACT

Real quaternions, given as an expansion of 4-dimensional complex numbers, were described in 1843 by the Irish mathematician Sir William Rowan Hamilton. The set of real quaternions forms a quaternion algebra. By describing quaternions, Hamilton introduced a new multiplication operation into vector algebra, so that division for two vectors is also possible. With the definition of a new product on the set of quaternions, the examination of motions in Euclidean space is facilitated. In this study, the algebraic structure of Split quaternions was constructed on the object obtained by using prime numbers and the properties of these types of quaternions were investigated.

***K*eywords** Quaternion · Quaternion Algebra · Split Quaternion Algebra

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GENERALIZED RIESZ POTENTIAL OPERATOR IN THE MODIFIED MORREY SPACES

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ABSTRACT

For a measurable function $\rho : (0,\infty) \to (0,\infty)$ the generalized Riesz potential operator I_ρ and the generalized fractional maximal operator M_ρ are defined by

$$I_\rho f(x) := \int_{\mathbb{R}^n} \frac{\rho(|x-y|)}{|x-y|^n} f(y)dy, \quad M_\rho f(x) := \sup_{r>0} \frac{\rho(r)}{r^n} \int_{B(x,r)} |f(y)|dy$$

for any suitable function f on $\mathbb{R}^n$, respectively. The generalized Riesz potential operator I_ρ was initilally investigated in [1]. The boundedness of I_ρ and M_ρ in the Morrey-type spaces is studied by many authors, for instance see [2, 3, 4, 5, 6, 5].

We denote by $\widetilde{L}_{p,\lambda}(\mathbb{R}^n)$ the modified Morrey space given in [2], as the set of locally integrable functions $f(x)$, $x \in \mathbb{R}^n$, with the finite norms

$$\|f\|_{\widetilde{L}_{p,\lambda}} := \sup_{x\in\mathbb{R}^n, t>0} [\min\{1,t\}]^{-\frac{\lambda}{p}} \|f\|_{L_p(B(x,t))}, \tag{1}$$

and if $\lambda < 0$ or $\lambda > n$, then $L_{p,\lambda}(\mathbb{R}^n) = \widetilde{L}_{p,\lambda} = \Theta$, where Θ is the set of all functions equivalent to 0 on $\mathbb{R}^n$.

We denote by $W\widetilde{L}_{p,\lambda}(\mathbb{R}^n)$ the modified weak Morrey space as the set of all locally integrable functions $f(x)$, $x \in \mathbb{R}^n$ with finite norms

$$\|f\|_{W\widetilde{L}_{p,\lambda}} := \sup_{x\in\mathbb{R}^n, t>0} [t]_1^{-\frac{\lambda}{p}} \|f\|_{WL_p(B(x,t))}.$$

In this talk, we prove the boundedness of the generalized fraction maximal operator M_ρ and the generalized Riesz potential operator I_ρ from the modified Morrey spaces $\widetilde{L}_{p,\lambda}(\mathbb{R}^n)$ to another one $\widetilde{L}_{q,\lambda}(\mathbb{R}^n)$, for $1 < p < q < \infty$ and from $\widetilde{L}_{1,\lambda}(\mathbb{R}^n)$ to the weak modified Morrey spaces $W\widetilde{L}_{q,\lambda}(\mathbb{R}^n)$, for $p = 1, 1 < q < \infty$. That is, in the following we give the sufficient conditions for the boundedness of the generalized Riesz potential in the modified Morrey spaces and modified weak Morrey spaces.

Theorem 1 *Let $0 \leq \lambda < n$, $1 \leq p < q < \infty$, the function ρ be a positive and measurable function and $f \in \widetilde{L}_{p,\lambda}(\mathbb{R}^n)$.*

(i) If $1 < p < q < \infty$ and ρ satisfies the condition $\rho(r) \leq Cr^{\frac{n}{p}-\frac{n}{q}}, C > 0$ then the generalized Riesz potential operator I_ρ is bounded from $\widetilde{L}_{p,\lambda}(\mathbb{R}^n)$ to $\widetilde{L}_{q,\lambda}(\mathbb{R}^n)$ and the following norm inequality satisfies, i.e.,

$$\|I_\rho f\|_{\widetilde{L}_{q,\lambda}} \lesssim \|f\|_{\widetilde{L}_{p,\lambda}}.$$

(ii) If $p = 1, 1 < q < \infty$ and ρ satisfies the condition $\rho(r) \leq Cr^{n-\frac{n}{q}}, C > 0$ then the generalized Riesz potential operator I_ρ is bounded from $\widetilde{L}_{1,\lambda}(\mathbb{R}^n)$ to $W\widetilde{L}_{q,\lambda}(\mathbb{R}^n)$ and the following norm inequality satisfies, i.e.,

$$\|I_\rho f\|_{W\widetilde{L}_{q,\lambda}} \lesssim \|f\|_{\widetilde{L}_{1,\lambda}}.$$

And also, in the following we give the sufficient conditions for the boundedness of the generalized fractional maximal operator in the modified Morrey spaces and modified weak Morrey spaces.

Theorem 2 *Let $0 \leq \lambda < n$, $1 \leq p < q < \infty$, the function ρ be a positive and measurable function and $f \in \widetilde{L}_{p,\lambda}(\mathbb{R}^n)$.*

(i) If $1 < p < q < \infty$ and ρ satisfies the condition $\rho(r) \leq Cr^{\frac{n}{p}-\frac{n}{q}}, C > 0$ then the generalized fractional maximal operator M_ρ is bounded from $\widetilde{L}_{p,\lambda}(\mathbb{R}^n)$ to $\widetilde{L}_{q,\lambda}(\mathbb{R}^n)$ and the following norm inequality satisfies, i.e.,

$$\|M_\rho f\|_{\widetilde{L}_{q,\lambda}} \lesssim \|f\|_{\widetilde{L}_{p,\lambda}}.$$

(ii) If $p = 1, 1 < q < \infty$ and ρ satisfies the condition $\rho(r) \leq Cr^{n-\frac{n}{q}}, C > 0$ then the generalized fractional maximal operator M_ρ is bounded from $\widetilde{L}_{1,\lambda}(\mathbb{R}^n)$ to $W\widetilde{L}_{q,\lambda}(\mathbb{R}^n)$ and the following norm inequality satisfies, i.e.,

$$\|M_\rho f\|_{W\widetilde{L}_{q,\lambda}} \lesssim \|f\|_{\widetilde{L}_{1,\lambda}}.$$

***K*eywords** Generalized Riesz potential operator · Generalized fractional maximal operator · Modified Morrey spaces

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SOME PROPERTIES OF HORADAM HYBRID QUATERNIONS

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ABSTRACT

In this study, we define the Horadam hybrid quaternions and give some of their properties. Moreover, we investigate the relations between the Fibonacci hybrid quaternions and the Lucas hybrid quaternions which connected to the Fibonacci quaternions and Lucas quaternions. Furthermore, we give the Binet formulas and Cassini identities for these hybrid quaternions. The hybrid quaternions have recently been defined by Dagdeviren as a new quaternion system. This system has a strong algebraic structure and it can be regarded as a generalization of complex, dual, and hyperbolic quaternions. Furthermore we will examine the Fibonacci and Lucas hybrid quaternions in detail and consequently we will give some properties and identities of these numbers.

***K*eywords** Fibonacci numbers · Hybrid numbers · Quaternions · Hybrid quaternions

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KOLMOGOROV ENTROPY OF MULTIPLIER OPERATORS ON TWO-POINT HOMOGENEOUS MANIFOLDS

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ABSTRACT

For a compact set $A \subset X$ in a Banach space X with the unit ball B_X we define Kolmogorov entropy number $e_n(A,X)$ as the infimum of all $\epsilon > 0$ such that there exist $\{x_k\}_{k=1}^{2^{n-1}} \subset X$ such that $A \subset \bigcup_{k=1}^{2^{n-1}} (x_k + \epsilon B_X)$. We present a new method of evaluation $e_n(A,X)$ which is based on volume estimates of John-Löwner ellipsoids. This approach allows to get sharp orders of entropy in the situations where known methods meet difficulties of a fundamental nature. In particular, in the case of two-point homogeneous Riemannian manifolds $\mathbb{M}^d$ (i.e. a real sphere $\mathbb{S}^d$, a complex $\mathrm{P}^d(\mathbb{C})$ or quaternion $\mathrm{P}^d(\mathbb{H})$ projective space or a Cayley elliptic plane $\mathrm{P}^{16}(\mathrm{Cay})$) we calculate sharp orders of entropy of standard Sobolev classes $W_\infty^\alpha\left(\mathbb{M}^d\right)$, in $L_1\left(\mathbb{M}^d\right)$,

$$e_n(W_\infty^\alpha\left(\mathbb{M}^d\right), L_1\left(\mathbb{M}^d\right)) \asymp n^{-\alpha/d}, \quad \alpha > 0.$$

Similar results obtained for multiplier operators on $\mathbb{M}^d$. Our results improve the estimate

$$n^{-\alpha/d}\log^{-1} n \ll e_n(W_\infty^\alpha\left(\mathbb{M}^d\right), L_1\left(\mathbb{M}^d\right)) \ll n^{-\alpha/d}.$$

obtained in [2], [1].

***K*eywords** Entropy · homogeneous manifold · Volume · John-Löwner ellipsoid

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NONLINEAR BEHAVIOR OF A MICRO-RESONATOR DESCRIBED BY A DUFFING TYPE OSCILLATOR

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ABSTRACT

Micro-Electromechanical Systems (MEMS) are developed for microtechnological applications [1] and also used for basic research [2][4]. MEMS opened up a whole new experimental window into the study of nonlinear dynamics of discrete systems in the form of nonlinear micromechanical oscillators and resonators.

The most notable MEMS are the microsensors and microactuators [3, 5]. They are categorized as "transducers" and defined as devices that convert energy from one form to another. The microsensor converts a measured mechanical signal into an electrical signal. Whereas a microactuator does vice versa [3].

Simple Harmonic Oscillators usually simulate mechanical resonators. That is usually a good approximation as most materials can sustain relatively large deformations before their intrinsic stress-strain relation breaks away from a simple linear description. Most evident are nonlinear effects that are mathematically described as a force proportional to the cube of the displacement. Such terms turn a simple harmonic resonator with a linear restoring force into a so-called Duffing resonator. Typical resonators include one movable electrode excited by DC and AC voltages applied on one or in both sides [6, 7].

We study an electrostatically actuated microresonator with electrostatic force on both sides. A Duffing-type oscillator simulates the horizontal displacement of the microresonator. Since a Duffing-type oscillator simulates the microresonator, the system, as expected, must have periodic oscillations with different periods, semi-periodical and also chaotic oscillations for different values of the parameters and initial conditions. The knowledge of the different oscillations is significant for constructing MEMS, its proper functioning, and its control.

Our work confirmed this rich dynamical behavior. We present a detailed investigation of the dynamics for different values of the excitation amplitude and the damping parameter. Also, we investigate that the kind of chaotical behavior is homoclinic.

***K*eywords** Micro mechanical resonator · Duffing type oscillators · Nonlinear oscillators · Chaotic dynamics · Homoclinic chaos

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VECTOR SPACES AND SOME APPLICATIONS

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ABSTRACT

The term "space" is not easy to understand and maybe considered something complicated. Moreover, in science, vectors can be considered as an arrow with a length and a direction. But in mathematics, the combination of these two words has a different meaning, i.e., a vector space is a set V with two operations: addition of vectors and scalar multiplication; these operations satisfy certain properties.

At this paper, we present some interesting and spectacular applications and illustrations of vector spaces.

Keywords Vector Space · Matrices · Hamming Distance

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SOLITON SOLUTION OF THE HIROTA EQUATION BY MODIFIED ANALYTICAL METHOD

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ABSTRACT

Nonlinear Schrodinger equation plays significant role in optical soliton communication. To improve transmission speed in optical soliton communications, high-power and ultra-short optical pulses should be used. And generalized third order nonlinear Schrödinger equation is used in optical fibres for describing ultra-short pulses. In this study, we utilized generalized Riccati mapping method to get numerous kinds of optical solitons of Hirota equation which is special generalized nonlinear Schrödinger equation of third order.

***K*eywords** Hirota equation · Generalized Riccati mapping method · Optical soliton

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CHIRPED W-SHAPED OPTICAL SOLITONS OF MODIFIED NONLINEAR SCHRÖDINGER EQUATION

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ABSTRACT

The aim of this paper investigated the exact nonlinearly chirped W-shaped soliton solutions of modified nonlinear Schrödinger equation which is proposed to describe the short pulse propagation in long monomode optical fibers in consideration of the inherent property of asymmetric output pulse spectrum. Firstly, we get the corresponding chirping parameter of the modified nonlinear Schrödinger equation by use of a complex envelope traveling-wave ansatz. Secondly, substituting this chirping parameter to the modified nonlinear Schrödinger equation has been reduced to an elliptic differential equation with a fourth-degree nonlinear term. Thirdly, we apply localized soliton ansatz of the sech type which allows for obtaining W-shaped soliton solution. Lastly, we get the 2-dim and 3-dim graphs of the W-shaped soliton solution by giving specific values to the parameters.

***K*eywords** Ansatz method · W-shaped soliton · Modified nonlinear Schrödinger equation

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BAYESIAN INFERENCE OF THE GENERALIZED LOG-LOGISTIC ACCELERATED FAILURE TIME MODEL FOR CENSORED SURVIVAL DATA

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ABSTRACT

The log-logistic, log-normal and Weibull distributions are extensively used to model survival data. The log-logistic and the log-normal families are used to model non-monotone (or unimodal) hazard rates, whereas the Weibull family is widely applied to model monotone hazard functions. The increasing availability of survival data with a wide range of characteristics motivate statisticians to develop more flexible parametric models that accommodate both monotone and nonmonotone hazard functions. One such model is the generalized log-logistic distribution which not only accommodates unimodal hazard functions but also allows for a monotone and bathtub shape hazard rates. This distribution has demonstrated considerable potential in univariate analysis of survival data. However, the primary focus of many studies is rather on understanding the relationship between the time to the occurrence of an event and one or more covariates. This leads to a consideration of regression models that can be formulated in different ways in survival and reliability analysis. One such strategy involves formulating models for the accelerated failure time family of distributions. The most commonly used distributions serving this purpose are the log-logistic, log-normal and Weibull distributions. In this paper, we show that the generalized log-logistic distribution is closed under the accelerated failure time family. We then formulate an accelerated failure time model based on the generalized log-logistic distribution Furthermore, we carry out the Bayesian approach and performance of Gibbs sampling using Markov chain Monte Carlo simulation study. A real-life data set based on censored information demonstrate that the generalized log-logistic accelerated failure time model can be valuable in adequately describing survival data with different hazard shapes.

***K*eywords** Bayesian inference · log-logistic distribution · Maximum likelihood Estimaton · accelerated failure time model · censored data · generalized log-logistic distribution

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ON THE HOLOMORPHIC CURVATURE TENSOR OF GENERALIZED COMPLEX SPACE FORMS

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ABSTRACT

In 1989, Z. Olszak has worked on the existence of a generalized complex space form. In 1998, M. Prvanović has introduced a tensor of Kaehler type for an almost Hermitian manifold, this tensor called holomorphic curvature tensor and reduces to the Riemannian curvature tensor R in an almost Kaehler manifold. Then M. Prvanović gave some properties about such tensor.

In this work, first we determine the holomorphic curvature tensor of generalized complex space forms and study some properties of this tensor in generalized complex space forms. Then it is proved that the difference tensor $(HR \cdot H(C) - H(C) \cdot HR)$ and the tensor $Q(H(S), H(C))$ of any generalized complex space form $M(f_1, f_2)$ of dimensional $(m \geq 4)$ are linearly dependent at every point of $M(f_1, f_2)$. Finally some results on generalized complex space forms satisfying curvature identities named Walker type identities are presented. It is proved that an m-dimensional $(m \geq 4)$ generalized complex space form satisfies Walker type identities.

***K*eywords** Generalized complex space form · Holomorphic curvature tensor · Walker type identity

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DISCRETE BIORTHOGONAL SYSTEMS AND EQUILIBRIUM CONDITION IN THE HARDY SPACE OF UNIT DISC AND UPPER HALF-PLANE

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ABSTRACT

In our presentation we start from the Malmquist-Takenaka complex orthonormal system. The Malmquist–Takenaka system is a system of rational functions–products of Blaschke factors–in the Hardy space of unit disc, which contains the classical trigonometric system, as special case. It is frequently applied in system identification in order to approximate the transfer functions. Discretization results connected to Malmquist-Takenaka systems for the unit disc and the upper half plane were published earlier in [3]. Based on these an analogue of discrete Fourier transform (DFT) was developed and the discrete versions was applied successfully for compression and representation of human ECG signals.

Discretization nodes on the unit circle and on the real line have similar properties: they satisfy some equilibrium conditions and are stationary points of some logarithmic potentials. The problem whether they are discrete energy minimizer configurations was formulated in the papers of Pap and Schipp [3] and was answered positively recently by Gaál, Nagy, Nagy-Csiha and Révész [2].

In a recent paper [1], Fridli and Schipp introduced the dual of the Malmquist-Takenaka system on the unit disc by using inversion, and proved discrete biorthogonal property on a set of points of the unit disc. Based on the presented results connected to Malmquist-Takenaka systems and the discrete orthogonality on the unit circle and real line, in this presentation we introduce the dual system of the Malmquist-Takenaka system on the upper half-plane and we prove discrete biorthogonality result on a set of discretization points on upper half-plane. We study the properties of discretization points on the disc and the upper half-plane, and we prove that they satisfy analogue equilibrium conditions like on the unit circle and the real line. We give a physical interpretation of the results, connected them with some logarithmic potentials.

***K*eywords** Hardy spaces · Malmquist-Takenaka systems · discrete biorthogonality · Blaschke products · equilibrium condition

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EXACT SOLUTIONS OF SOME IMPORTANT NONLINEAR FRACTIONAL PARTIAL DIFFERENTIAL EQUATIONS BY THE F-EXPANSION METHOD

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ABSTRACT

In this study, we give an analysis by using the F-expansion method to create exact solutions of the space-time fractional modified Benjamin Bona Mahony equation and the nonlinear time fractional Schrödinger equation with beta derivative. The F-expansion method is one of the effective and important methods for finding exact solutions of nonlinear equations. More exact solutions described by the Jacobi elliptic function have been found with the help of Maple. Hyperbolic function solutions and some exact solutions expressed by trigonometric functions have been obtained in the case of m modulus 1 and 0 limits of the Jacobi elliptic function.

***K*eywords** Modified Benjamin Bona Mahony equation · Schrödinger equation · Beta derivative · F-expansion method

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Almost Metallic Structures on Manifolds with Linear Connection

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Abstract

The metallic ratio, which is a generalization of the golden proposition was introduced in 1999 by de Spinadel [7] and named as the metallic means family or metallic propositions. In [2, 3], the authors defined almost metallic manifolds with the help of these propositions and investigated the geometry of these manifolds.

In the present study, we investigate Schouten and Vranceanu connections with respect to which an almost metallic structure is parallel. Then, the integrability conditions of the almost metallic structures.

***K*eywords** Almost metallic structure · Metallic structure · Schouten connection · Vrănceanu connection · Parallelism · Half parallelism · Anti half parallelism · Integrability · Tangent bundle

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ON BRONZE MANIFOLDS

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ABSTRACT

It is well known defined that many polynomial structures on a differentiable manifold with the help of by J which are $C^\infty-$tensor fields of type $(1,1)$. Almost tangent structures, almost product structures and almost complex structures, golden structures, silver structures and metallic structures are some examples. These structures have been recently studied by many authors (see [1, 2, 3, 6, 8, 9]).

The metallic ratio, which is a generalization of the golden proposition was introduced in 1999 by de Spinadel [7] and named as the metallic means family or metallic propositions. A member of this metallic ratio family, the Bronze ratio is defined in [5]. In [7], Kalia introduced a new Bronze ratio definition that is not a member of this family. In [10], Şahin defined almost poly-Norden manifolds with the help of this new bronze ratio, and investigated the geometry of these manifolds.

In this study, by using the new bronze mean, we investigate new almost bronze structures, i.e. polynomial structures with the structure polynomial $Q(J) = J^2 - mJ + I$, $m \in \mathbb{R} - [-2, 2]$ on manifolds. We study for integrability and parallelism conditions of almost bronze structures. Also, we obtain a bronze Riemannian manifold with respect to the Riemannian metric.

***K*eywords** Bronze mean · Almost bronze structure · Almost bronze manifold · Polynomial structure

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METALLIC ANTI-KÄHLER MANIFOLDS

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ABSTRACT

Let p and q be two positive integers. The positive roots of the equation $x^2 - px - q = 0$ is named member of the metallic means family, which was introduced by Spinadel in [7]. These roots, denoted by $\sigma_{p,q} = \frac{p+\sqrt{p^2+4q}}{2}$ are also called $(p,q)-$ metallic numbers. In [2, 3], the authors defined metallic manifolds with the help of these numbers and investigated the geometry of these manifolds. In [6], the authors have introduced complex metallic means family $\sigma^c_{p,q} = \frac{p+\sqrt{p^2+4q}}{2}$ which is complex root of $x^2 - px + \frac{3}{2}q = 0$, where p and q are real numbers satisfying $q \geq 0$ and $-\sqrt{6q} < p < \sqrt{6q}$. The same authors have also introduced almost complex metallic structures by inspiring $\sigma^c_{p,q}$.

Let M^{2k} be a $2k-$dim manifold and J_M be a tensor fields of type $(1,1)$ on M^{2k}. An almost complex metallic manifold (M^{2k}, J_M) equipped with a Norden metric is called an almost metallic Norden manifold.

The study is devoted to the study of metallic anti-Kähler structures on metallic Norden manifolds, and some properties of these manifolds are examined. In addition, various properties of these manifolds were examined.

***K*eywords** Anti-Kähler manifold · Norden metric · Linear connection · Almost complex metallic structure · Metallic structure

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NEUTROSOPHIC SOFT e-OPEN MAPS, NEUTROSOPHIC SOFT e-CLOSED MAPS AND NEUTROSOPHIC SOFT e-HOMEOMORPHISMS IN NEUTROSOPHIC SOFT TOPOLOGICAL SPACES

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ABSTRACT

In this article, we introduce the concept of NSe-open and NSe-closed mappings in neutrosophic soft topological spaces and study some of their related properties. Further the work is extended to NS homeomorphism, NSe-homeomorphism, NSe-C homeomorphism and $NSeT_{\frac{1}{2}}$-space in neutrosophic soft topological spaces and establish some of their related attributes.

***K*eywords** NSe-open map · NSe-closed map · NSe-homeomorphism · $NSeT_{\frac{1}{2}}$-space · NSe-C homeomorphism

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ICMASE 2021

PQ-CALCULUS OF FIBONACCI DIVISORS AND METHOD OF IMAGES IN PLANAR HYDRODYNAMICS

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ABSTRACT

Division of two Fibonacci numbers F_{nk}/F_k is an integer number, which we call as Fibonacci divisor $F_n^{(k)}$, conjugate to F_k. The Fibonacci divisor numbers are determined by the Binet formula for pq-numbers, $F_n^{(k)} = [n]_{\varphi^k,\varphi'^k}$, with basis numbers as powers of the Golden Ratio φ, φ'. By introducing the hierarchy of Fibonacci divisor derivatives $D_F^z = F_{z\frac{d}{dz}}^{(k)}$ [1], we develop corresponding calculus and derive several identities, such as addition rules and sum of Fibonacci divisors, hierarchy of Golden binomials, corresponding hierarchy of exponential functions, translation operator and infinite hierarchy of Golden analytic functions. This calculus generalizes for arbitrary k, the Golden calculus developed in [2] and corresponding to $k = 1$. The hierarchy of Golden periodic functions $D_F^z f(z) = 0$, appearing in this calculus we relate with the method of images in planar hydrodynamics for incompressible and irrotational flow [3]. We show that the even and the odd hierarchy of these functions, determine the flow in bounded domain, according to the two circle theorem [4] and the double-circular wedge theorem [5], correspondingly. In the first case we have flow in the annular domain, bounded by concentric circles with the Golden Ratio of radiuses. And in the second case, the problem is in the wedge domain bounded by two circles in the Golden proportion. As an example, complex potential and velocity field for the set of point vortices in corresponding domains are calculated explicitly.

***K*eywords** Fibonacci divisors · Golden Calculus · Hydrodynamic Images

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STATISTICALLY CONVERGENT SEQUENCES IN INTUITIONISTIC FUZZY METRIC SPACES

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ABSTRACT

In 2004, Park [5] defined the concept of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space [4]. A lot of developments have been studied on fuzzy metric spaces and intuitionistic fuzzy metric spaces. The notions of statistical convergence was introduced by Fast [3] and Steinhous [6] independently, and this idea draw attention both of mathematicians working on the field of pure and applied mathematics. In 2020, Changqing et. al. [2] studied on statistically convergent sequences in fuzzy metric spaces. In this paper, we introduce statistical convergence on intuitionistic fuzzy metric spaces and analyze relations of convergence and statistical convergence on intuitionistic fuzzy metric spaces. Further, we study statistical Cauchy sequences and statistical completeness on intuitionistic fuzzy metric spaces.

***K*eywords** intuitionistic fuzzy metric · statistical convergence · statistical Cauchy · statistical completeness

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DIFFERENT TIME SCHEMES WITH DIFFERENTIAL QUADRATURE METHOD IN CONVECTION-DIFFUSION-REACTION EQUATIONS

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ABSTRACT

In this study, different time schemes are investigated in numerical simulation of two-dimensional, unsteady convection-diffusion and convection-diffusion-reaction equations. Convection-diffusion equations, also adding reaction term, describe many physical problems such as heat transfer, chemical reaction processes, fluid dynamics etc. In order to be able to interpret the physical reality as much as possible, the numerical simulations should be executed not only as accurate as possible but also computationally efficient in view of less memory usage and short period of time of central processing unit (CPU) in computers. In this study, the space derivatives are approximated by differential quadrature method which gives highly accurate results using small number of grid points. For an unsteady problem, the accuracy is also affected by the approximation of time derivatives. Therefore, different time schemes are examined in this study. Implicit time schemes such as the first, the second and the third order backward differentiation formulas, trapezoidal rule and the third order Adams-Moulton formula are performed as well as explicit methods such as the second order predictor-corrector method, the fourth order Adams-Moulton-Bashforth formula, Runge Kutta of order four, three stages strong stability preserving Runge Kutta of order three and five stages strong stability preserving Runge Kutta of order four. Most of these time schemes are embedded into ordinary differential equations. Now, these are formulized for governing partial differential equations of current problem. The distribution of nodes is chosen as non-uniform Gauss-Chebyshev-Lobatto grid points since non-uniform grids provides more accurate solutions than the equally spaced grids. The problems having the exact solutions are chosen to check the best error behavior. Absolute maximum error, relative error, and average absolute maximum error at the required time level are compared between the exact solution and the approximated solutions. Computational cost in view of CPU times and the efficiency of different time schemes in view of errors are examined. As expected, explicit time schemes need smaller time

increments while implicit time schemes enable one to use larger time increments. The second and the third order backward differentiation formulas are more reliable due to their unconditionally stable nature comparing with the other time schemes in the chosen problems.

***K*eywords** convection-diffusion-reaction · differential quadrature method · time schemes

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ONSITE AND/OR ONLINE ASSESSMENT?

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ABSTRACT

During the world pandemic of Covid 19, the need to transform teaching and learning methods was urgent and their impact on evaluation was also felt. This paper reflects the necessary adjustments on the assessment that were proposed to students during the academic year of 2020/2021, always having in mind a set of competencies already recognized as essential for engineering student graduates,([1], [2] and [4]). Learning from or based upon a real problem is an idea that gathered in former years more and more teacher's agreement. In fact, we all seem to agree that this is the way for students to get acquainted with the importance of the contents that are taught and how they may be applied in the real world. Nevertheless, for students to solve the proposed problems from beginning to end is a hard task that must be tutored closely. To assess the competencies acquired with this type of teaching and learning using online tools when the number of students is high ($>$ 100) is a challenge. On 2020/2021 all classes were given online, and the assessment was performed in a different way. Students were asked to solve multiple choice question(s) at each class as a way of keeping them focused on the contents that were being presented and to improve students' participation. At the end of the semester a test was performed, and their final grade was the summation of the continuous assessment and the test grade. Conceptually the questions, either of continuous assessment or test, were designed observing the principles followed in the previous years (separating as much as possible the competencies that were being evaluated) as a continuation of Rules_Math project implementation. One major issue with online assessment was the high possibility of cheating those students had, since no proctoring was made to their screens and, due to the high number of students at each class and internet difficulties, some of them had moments where their cameras and microphones were not turned on. This paper presents an experience and its results when compared to the ones obtained when the assessment was done exclusively on site. From the results, conclusions are drawn especially regarding learning results obtained within this process. The results presented also take into account students background, namely if students come from regular secondary courses or from professional courses. The author believes that although there exists an impossibility of

giving continuous individual attention, the student center teaching and learning process together with competence-based methodology and a previous Index of Learning Styles, ([3],[4] and [5]), students will certainly increase student's assessment results and their in-market performance.

***K*eywords** Online Assessment · Mathematics Competencies · Teaching and Learning Methods

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A MODIFIED LESLIE-GOWER TYPE PREDATION MODEL CONSIDERING ALLEE EFFECT ON PREY AND COMPETENCE AMONG PREDATORS.

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ABSTRACT

The ecological theory is a rich area for mathematical modeling. A variety of mathematical approaches and frameworks are been provided for a better understanding of the populational dynamics.

Particularly, the dynamic relationship between predators and their prey has been and will continue to be one of the dominant themes in this area. This is due to its universal existence and because it allows a better understanding of the behavior of food chains or food webs.

It is well known that predator-prey interactions are strongly dependent on the functional response as well as the population growth rates of prey and predators.

In this work, the analysis will be based on a Leslie-Gower type predation model, described by a two-dimensional system of ordinary differential equations (ODEs), assuming that the prey population is affected by an Allee effect, which modifies the usual logistic equation.

The functional response will be assumed linear, which is prey-dependent and monotonously increasing. In turn, the equation of growth of predators will also be considered of the like-logistic type, where the environmental carrying capacity for predators is assumed proportional to the prey population size.

Among the most important results obtained is that for the same set of parameters, there are different behaviors of the system solutions, since two attractor singularities can appear simultaneously. Then, populations can coexist around fixed population sizes, or the prey population can become extinct.

We estimate that the analytical results obtained have an adequate ecological interpretation, under the underlying assumptions in the modeling with ODEs.

Numerical simulations are given to endorse the analytical results and to exhibit the richness of the dynamics in the system.

***K*eywords** Bifurcation · stability · limit cycle · predator-prey model · functional response.

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MINIMAL TRANSLATION LIGHTLIKE GRAPHS IN SEMI-EUCLIDEAN SPACE

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ABSTRACT

In this paper we study the translation lightlike graphs which are generalization of the translation lightlike hypersurfaces in semi-Euclidean space. We prove that all translation lightlike graphs are locally the hyperplanes. According to this fact, translation lightlike graphs are minimal.

***K*eywords** Degenerate surfaces · Minimality · Semi-Euclidean geometry

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On The Dual Spherical Indicatricies of Dual Slant Helices in $\mathbb{D}^3$

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Abstract

In this paper, we characterize dual slant helices in $\mathbb{D}^3$. In particular, we obtain the relations between a dual general helix and a dual slant helix.

***K*eywords** Dual curves · Dual space

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NUMERICAL SIMULATION OF THE IMPROVED KARDAR-PARISI-ZHANG DISCRETIZATION EQUATION WITH GAUSSIAN NOISE TERM

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ABSTRACT

One of the most important models for studying growth of surface is the Kardar-Parisi-Zhang (KPZ) equation. It shows the rate of a profile $h(x,t)$ at substrate position x and time t. [1]:

$$\frac{\partial h(x,t)}{\partial t} = v\nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(x,t), \tag{1}$$

where v and λ are the diffusion coefficient and the nonlinear parameter, respectively. Here the noise term η has a Gausian distribution and it means zero correlation. Because of some implicit lower cutoff, the profile $h(x,t)$ is assumed as a course grained.

The most appropriate approach for the investigation of KPZ equation is direct numerical integration. Till now there most studies based on the discrete equation or simple variants of it [5-7]. Also there most studies carried out on the paramaters of D, λ and v but it was not enough to prove the real relationship between the universality of discretization method equation (2) and KPZ equation [2].

$$h_i^{n+1} = h_i^n + \Delta t\left[v\left(h_{i+1}^n + h_{i-1}^n - 2h_i^n\right) + \frac{\lambda}{8}(h_{i+1}^n - h_{i-1}^n)^2\right] + \sqrt{2D\Delta t}\xi_i^n \tag{2}$$

Later by the authors [2] suggested a scheme involving on spatial discretization denoted symbolically as equation (3) and the equation (2) results from both spatial and temporal discretization of the KPZ equation (1). Due to lack of the explanation on connection between equations (1) and (**??**), it was instructive to study the following improved novel discretization:

$$\frac{dh_i(t)}{dt} = v_0\Gamma_i + \frac{\lambda_0}{2}\psi_i + \eta_i(t), \tag{3}$$

for $i = 1$ to L with periodic boundary conditions, where

$$\Gamma_i = h_{i+1} + h_{i-1} + 2h_i \tag{4}$$

$$\psi_i = \left(\frac{1}{3}\right)\left[(h_{i+1} - h_i)^2 + (h_{i+1} - h_i)(h_i - h_{i-1}) + (h_i - h_{i-1})^2\right]. \tag{5}$$

The Gaussian distribution noise $\eta_i(t)$ and

$$< \eta_i(t)\eta_j(t') >= 2D_0\delta_{ij}\delta(t - t'). \tag{6}$$

When the interfaces were smooth, both equations (2) and (3) could be equally valid for the equation (1) [7].

After improved discritizition Kardar-Parisi-Zhang (KPZ) equation in 1+1 dimentions proposed by authors [2] with the noise η which equals to zero mean and a correlator. Then, we numerically simulated this improved discretization method of the KPZ equation in 1 + 1 dimensions with Gaussian noise term. While simulating the result we applied different amplitudes for our noise term. The amplitudes of a value are in between 0.1 to 2.5 which represent more accurate effect points on the surface. These amplitude differences showed behiviour of the noise term effects on the formation of the KPZ equation. In addition, during the simulation Bounary condition and Initial conditions are used as fixed points which has crucial role on the formation of surface evolution. Keeping them in stable numbers provided us possibility to analize the importance of noise term amplitude. Here, our initial condition equals to zero and it did not make difference to the expected result of our numerical simulation.

***K*eywords** Discretization method · Gaussian term · Amplitude effect · Kardar-Parisi-Zhang equation · explicit method

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DO ENERGY AND ECONOMIC GROWTH CONTRIBUTE TO ENVIRONMENTAL DEGRADATION? EMPIRICAL EVIDENCE FROM SELECTED EUROPEAN COUNTRIES

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ABSTRACT

Many countries are suffering from environmental degradation because of their rapid development. Energy is necessary for economic growth in most countries. However, numerous previous researches have suggested that increased energy use may have an impact on environmental quality. This study investigates relationships of energy use and economic growth to environmental degradation for a panel of 25 selected European (EU) countries from 2000 to 2019. Specifically, this study aims to determine whether neighboring countries influence pollution in a country and identify the variables affecting pollution among the EU countries. Carbon dioxide (CO_2) emission, gross domestic product (GDP) per capita and energy consumption per capita are used as proxies in the analysis. Besides ordinary least square regression (OLS), considering the influence of neighboring countries, we employ a spatial model to measure the spatial dependence effect in the region. The exploratory analysis using global (Moran's I) and local indication of spatial autocorrelation (LISA) suggested positive autocorrelation in all variables. The diagnostic tests detect a spatial dependence in the OLS, indicating that OLS regression is biased to model the data set. There are three spatial models considered: spatial autoregressive (SAR) model, spatial error (SEM) model and spatial Durbin (SDM) model. The LM test shows that SDM is the most appropriate to model the relationship for the dataset. The SDM estimated results indicate that economic growth significantly affects CO_2 emissions in the EU. In addition, the increase of CO_2 emissions in adjacent countries will increase the local CO_2 emissions through spatial spillover effects. Based on the findings, the results are consistent with most of the past studies and reveal useful information about the spatial impact on the nearby countries. Thus, this study provides a better understanding of the inter-relationship among the variables in developed countries like the EU to attain sustainable development. Although

this study focuses on CO_2 emissions, similar method can be used by using other primary pollutants, such as particulate matter, sulfur dioxide and nitrogen oxides, from a spatial viewpoint.

***K*eywords** CO_2 · Energy · Economic · Panel · Spatial

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DETERMINANTS OF INFLATION IN MALAYSIA: MONETARY OR REAL FACTOR?

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ABSTRACT

In this study, the analysis is performed to find the main determinants of inflation in Malaysia. In particular, the main focus is to reveal if inflationary is due to monetary factor or real factor. For this purpose, the inflation is assumed to behave as modelled by the Quantity Theory of Money (QTM) and the effect of monetary on inflation in Malaysia is examined. The data are in quarterly and range from year 1997 to 2018. The main determinants to be tested consist of money supply (M1, M2 and M3), real gross domestic product (RGDP) and real broad effective exchange rate (RBEER). The examination is based on two forms (log level and growth rate). The Autoregressive Distributed Lag (ARDL) model and Non-linear Autoregressive Distributed Lag (NARDL) model are employed. The examination is based on two forms: the log level and growth rate of inflation. The empirical results detect M3 as the best proxy for money supply in both cases with asymmetric relationship best fits for log level and symmetric relationship. The NARDL model also provides more inflation on the asymmetric effects of money supply increases and decreases on inflation in addition to the short-run and long-run estimated effects. The results reveal that money supply has short-run impact on inflation meanwhile the RGDP has both short-run and long-run impacts on inflation. There is no one-to-one relationship between them, hence violating QTM's condition. The relationship between money supply and inflation is less than one in the growth rate best model (symmetric), indicating QTM holds partially meanwhile in log level best model (asymmetric), QTM is invalid. On the other hand, the impact of RBEER is very limited. The study concludes that the main determinants of inflation are money supply and RGDP. Therefore, the policymakers should strive for stable economy growth through accommodation of both fiscal and monetary policy.

Keywords Inflation · Asymmetric effect · Quantity theory of money · Real factor

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NANO Z SEPERATION AXIOMS

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ABSTRACT

In this paper, the basic properties of $\mathfrak{N}ZT_0$, $\mathfrak{N}ZT_1$, $\mathfrak{N}ZT_2$, $\mathfrak{N}Zreg$ and $\mathfrak{N}Znor$ spaces are established. Further, the study of nano Z compactness and nano Z connectedness and also dealt with it.

***K*eywords** $\mathfrak{N}ZT_0$, $\mathfrak{N}ZT_1$, $\mathfrak{N}ZT_2$, $\mathfrak{N}Zreg$, $\mathfrak{N}Znor$ spaces, nano Z compactness and nano Z connectedness

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A MATHEMATICAL MODEL FOR PALLET OPTIMIZATION IN PARQUET COMPANY

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ABSTRACT

Product shipping is an issue of great importance in today's trade. Many transportation vehicles and auxiliary materials are used during transportation. One of them is the pallet and it is a product that can be found almost everywhere. In addition, companies are working in this direction, aiming to use multiple times instead of single-use pallets. A pallet is a flat transport structure used as a base, which assists goods in a stable fashion while being lifted by a forklift, a front loader, a jacking device, or an erect crane. In many studies, it was seen that the reuse of pallets with reverse logistics provides benefits in more than one way. In addition to reducing the total cost in the long run, it has also prevented environmental pollution and tree cutting. In this study, the current situation of a company producing parquet regarding the use of pallets was examined and poured into a mathematical model. The current state mathematical model has been solved with mixed integer linear programming model (MILP) in GAMS optimization program. Linear programming maximizes (or minimizes) a linear objective function subject to one or more constraints. Mixed integer programming adds one additional condition that at least one of the variables can only take on integer values. For the current situation, the total cost is minimized by the optimum number of pallets. Repairing and reusing the wooden pallets used to reduce the total cost is a solution suggested at the first stage. Another suggestion is the use of plastic pallets, which give less scrap and have higher durability. All three cases formed by optimization mathematical models have been solved separately with the GAMS program. Moving from the current model to the plastic pallet mathematical model, there has been a decrease in both the total cost and the amount of new pallets purchased each month.

Keywords Pallet · GAMS · Cost · Mathematical Model

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NEW RESULTS FOR BERNSTEIN OPERATORS

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ABSTRACT

Bernstein operators introduced by [1] in 1912 as follows:

$$B_s(v;x)=\sum_{r=0}^{s}\left(\begin{array}{c} s \\ r \end{array}\right)x^s(1-x)^{s-r}v\left(\frac{r}{s}\right),\quad s\geq 1$$

for $v\in C[0,1]$ and $x\in[0,1]$. In 2020, Usta [3] defined a modification of Bernstein operators for $v\in C(0,1)$, $s\in\mathbb{N}$ and $x\in(0,1)$ by

$$B_s^*(v;x)=\frac{1}{s}\sum_{m=0}^{s}\left(\begin{array}{c} s \\ m \end{array}\right)(m-sx)^2x^{m-1}(1-x)^{s-m-1}v\left(\frac{m}{s}\right).$$

In this talk, we mention some generalizations of the $B_s^*(v;x)$ operators. We investigate the rate of convergence by means of modulus of continuity and Peetre-$\mathcal{K}$ functionals. After that, we give Voronovskaya type asymptotic formula.

Keywords Modulus of continuity · Peetre-$\mathcal{K}$ functionals · Voronovskaya-type theorem

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ON MATHEMATICAL MODELLING OF HUMAN PHONATION PROCESS: FINITE ELEMENT APPROXIMATION OF FLOW INDUCED VOCAL FOLDS VIBRATIONS

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ABSTRACT

The fluid-structure-acoustic interaction problems are usually associated with technical applications as aeroelasticity. However, couplings between fluid flow, elastic structure deformation and acoustics are involved also in biomechanics of voice. Voice production is a complex process. It involves airflow induced vibrations of vocal folds, which generate a source sound. This source is further modified by the acoustic resonances in the vocal tract cavities. The vocal folds can start to oscillate at the so-called phonation onset (flutter instability) given by certain airflow rate and a certain prephonatory vocal folds position, see [1]. For higher flow rates, the glottis is closing during VFs vibration and the VFs collide loading the tissue periodically by the contact stress.

Thus this process is challenging task to be mathematically modelled which addresses flow field, structure deformation as well as acoustics, see e.g. [2]. Here, we focus on the mathematical modelling of the voice production, where the vocal folds contact phenomena is addressed. Because in voice the airflow velocity in the human glottal region is lower than 100 m/s, one can use separately the incompressible Navier-Stokes model for the fluid flow and the Lighthill's acoustic analogy for the acoustic wave propagation. In this paper a simplified problem is considered, mathematically described and numerically discretized using the finite element method with non-standard boundary conditions. The main attention is paid to the possible closure of the glottis, which is included in the model with the concept of a fictitious porous media and using the Hertz impact force The time dependent computational domain is treated with the aid of the Arbitrary Lagrangian-Eulerian method and the fluid motion is described by the incompressible Navier-Stokes equations coupled to the structural dynamics. In order to overcome the instability caused by the dominating convection due to high Reynolds numbers, stabilization procedures are applied

and numerically analyzed for a simplified problem. The possible distortion of the computational mesh is considered. Numerical results are shown and discussed.

***K*eywords** aeroelasticity · Navier-Stokes equations · Arbitrary Lagrangian-Eulerian method

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POSITIVE SOLUTIONS FOR CONCAVE-CONVEX ELLIPTIC PROBLEMS INVOLVING $p(x)$-LAPLACIAN

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ABSTRACT

In this work, we study the existence and nonexistence of positive solutions of the following nonlinear equation

$$-\Delta_{p(x)}u = \lambda\, k(x)u^q \pm h(x)u^r \quad \text{in} \quad \Omega, \quad u = 0 \quad \text{on } \partial\Omega, \qquad (Q)$$

where $\Omega \subset \mathbf{R}^N$, $N \geq 2$, a regular bounded open domain in $\mathbf{R}^N$ and the $p(x)$-laplacian

$$\Delta_{p(x)}u := \text{div}\left(|\nabla u|^{p(x)-2}\nabla u\right),$$

for a continuous function $p(x) > 1$ defined on Ω. The positive parameter λ induce the bifurcation phenomena. The study of the equation (Q) needs generalized Lebesgue and Sobolev spaces. In this paper, under suitable assumptions, we prove that some variational methods still work. We use them to prove the existence of positive solutions to the problem (Q) in $W_0^{1,p(x)}(\Omega)$. When we prove the existence of minimal solution, we use sub-super solutions method.

***K*eywords** Variable exponent Sobolev spaces, $p(x)$-Laplace operator, convex-concave nonlinearities, Variational methods.

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Fuzzy θ^*S-open and Closed Mappings in Sostak's Fuzzy Topological Spaces

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ABSTRACT

We introduce and investigate some new classes of mappings called fuzzy θ^*S-open map and fuzzy θ^*S- closed map to the fuzzy topological spaces in Sostaks sense. Also, some of their fundamental properties are studied. Moreover, we investigate the relationships between some other existing mappings.

***K*eywords** Fuzzy open, fuzzy θ^*-semiopen mappings, $f\theta^*S$ -T_1 space, $f\theta^*S$ -T_2 space, fuzzy

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FINITE ELEMENT APPROXIMATION OF COUPLED VIBRO-ACOUSTIC PROBLEM MOTIVATED BY PHONATION INTO TUBES

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ABSTRACT

The acoustic resonance frequencies of human vocal tract co-determine the dominant frequencies produced by the whole process of human phonation, [3]. In many circumstances the source-filter theory can be applied postulating that vocal tract acts a linear acoustic filter without backward influence on the sound production mechanism, see [4]. Nevertheless in the case of phonation into a tube when vocal tract acoustic resonance frequencies are close to the vocal folds mechanical eigenfrequencies a more complex model of vocal tract acoustics coupled to vocal folds vibrations needs to be considered, [1].

Here, the vocal folds vibrations are described using linear elasticity theory and the Helmholtz equation is used for frequency characterization of acoustic waves propagation in the vocal tract model. The both subproblems are numerically approximated by finite element method and the standard coupling conditions are used, see [2].

The preliminary results compare the acoustic resonance frequencies of vocal tract alone together with the resonance frequencies of the coupled vibro-acoustic system. The associated modal shapes are shown.

***K*eywords** Finite element method · Vibroacoustics · Helmholtz equation · Resonances of coupled system

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Acknowledgment

The financial support of this study was provided by the Czech Science Foundation under the Grant No. GA 19-04477S and by the Grant No. SGS19/154/OHK2/3T/12 of the Grant Agency of the CTU in Prague.

ON THE NICKEL FIBONACCI NUMBERS

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ABSTRACT

In this article, we focus on the second and the third order Nickel Fibonacci sequences. We give some properties they have, and we obtain some well known identities for them such as Binet's identity, Cassini(Simson) identity, De Moivre-type identity. Also, we examine the matrix representations and give some identities of the Nickel Fibonacci matrices. Additionally, we develop an encryption/decryption algorithm, in which we use Nickel Fibonacci matrices. Finally, we give two illustrative numerical examples that we encrypt and decrypt two plain texts with this algorithm.

Keywords De Moivre-type identity · Nickel Fibonacci numbers · Generating functions · Binet's formula

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ON FIXED POINT RESULTS FOR MIXED NONEXPANSIVE MAPPINGS

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ABSTRACT

Let (X, d) be a metric space, and K be a nonempty subset of X. We say that T is:
(i) said to satisfy condition (C) if

$$\frac{1}{2}d(x, Tx) \leq d(x, y) \text{ implies } d(Tx, Ty) \leq d(x, y),$$

for all $x, y \in K$,
(ii) α- nonexpansive if

$$d(Tx, Ty)^2 \leq \alpha d(Tx, y)^2 + \alpha d(x, Ty)^2 + (1 - 2\alpha)\, d(x, y)^2$$

for all $x, y \in K$ and for some $\alpha < 1$. In this work, we introduce an iterative process for a mapping satisfying condition (C) and an α- nonexpansive mapping in a convex metric space. We also prove that some fixed point theorems on strong convergence and Δ-convergence to common fixed point of such mappings.

***K*eywords** Iterative process · Convex metric space · Condition (C) · α- nonexpansive mapping

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